CHAPTER 2
PHOTONS AND ATOMS

Max Planck
Nobel prize 1918

Albert Einstein
Nobel prize 1921

1. Atoms, Molecules, and Solids

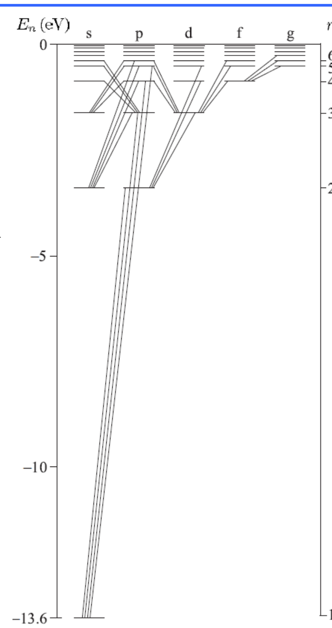
- Atomic Physics' starting point: the old Bohr model
  Schrodinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r},t)+V(\vec{r},t)\Psi(\vec{r},t)=j\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t}$$

Time independent Schrodinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r})+V(\vec{r})\psi(\vec{r})=E\psi(\vec{r})$$

-on discrete stationary states
-radiative transitions
quantum jumps between levels
-The old Bohr model
-energy levels →



For molecular oscillation:

$$E_q=(q+\frac{1}{2})\hbar\omega$$

Vibrational and Rotational Energy Levels of Molecules

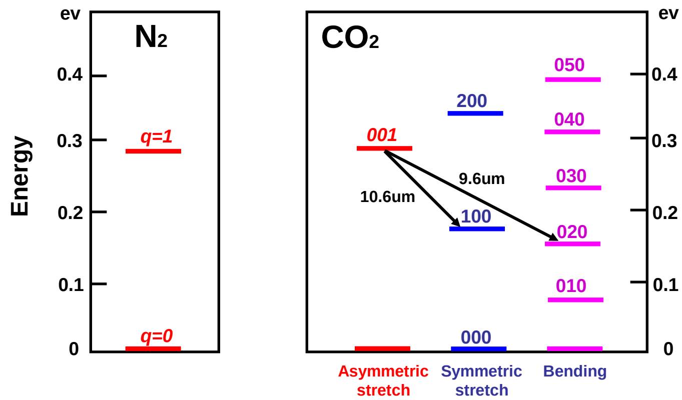


Figure Lowest vibrational energy levels of the N2 and CO2 molecules (the zero of energy is chosen at q=0). The transitions marked by arrows represent energy exchanges corresponding to photons of wavelengths 10.6um and 9.6um, as indicated. These transitions are used in CO2 lasers.

Electron Energy Levels of Atoms and Molecules

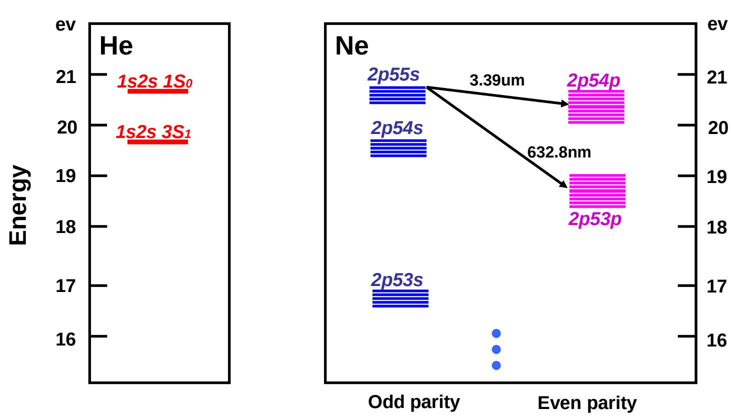


Figure 4.3-3 Some energy levels of He and Ne atoms. The He transitions marked by arrows correspond to photons of wavelengths 3.39m and 632.8nm, as indicated. These transitions are used in He-Ne lasers.

Electron Energy Levels in Solids

Isolated atoms and molecules exhibit discrete energy levels, as shown in Figs.4.3-5 to 4.3-8. For solids, however, the atoms, ions, or molecules in close proximity to each other and cannot therefore be considered as simple collections of isolated atoms; rather, they must be treated as a many-body system.

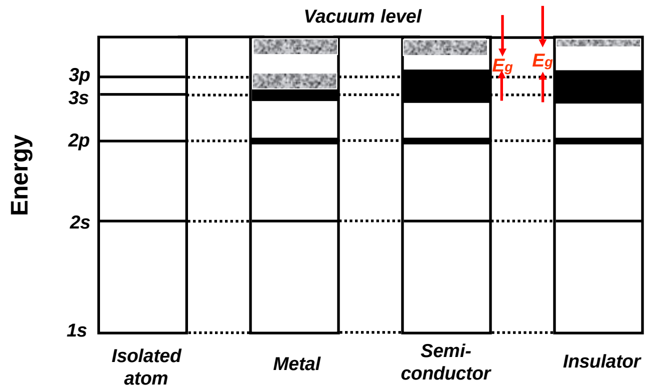


Figure 4.3-5 Broadening of the discrete energy levels of an isolated atom into bands for solis-state materials.

Quantum-well Structure

Multi-layers of different semiconductor materials

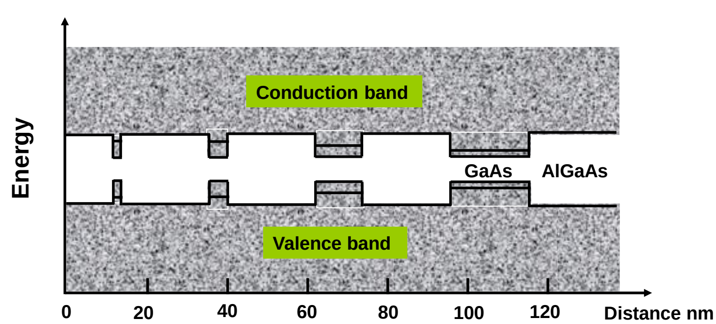
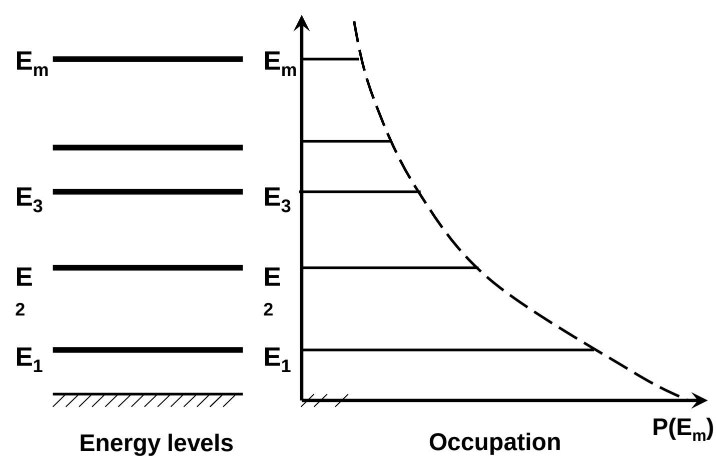


Figure 4.3-8 Quantized energies in a single-crystal AlGaAs/GaAs multiquantum-well structure. The well widths can be arbitrary (as shown) or periodic.

2. Occupation of energy levels in thermal equilibrium

Boltzmann distribution

$$P(E_m) \propto \exp(-E_m / k_B T), m = 1, 2, ...,$$



$$\frac{N_2}{N_1} = \exp(-\frac{E_2 - E_1}{k_B T})$$

The Boltzmann distribution depends on the temperature T. At $T = 0$ K, all atoms are in the lowest energy level (ground state). As the temperature increases the populations of the higher energy levels increase. Under equilibrium conditions, the population of a given energy level is always greater than that of a higher-lying level. This does not necessarily hold under nonequilibrium conditions, however. A higher energy level can have a greater population than a lower energy level. This condition, which is called a **population inversion**, provides the basis for laser action (see Chaps. 13 and 14).

$$\frac{N_2}{N_1} = \frac{g_2}{g_1}\exp(-\frac{E_2 - E_1}{k_B T})$$

Fermi-Dirac Distribution

Pauli exclusion principle

Electrons in a semiconductor obey a different occupation law. Since the atoms are located in close proximity to each other, the material must be treated as a single system within which the electrons are shared. A very large number of energy levels exist, forming bands. Because of the **Pauli exclusion principle**, each state can be occupied by at most one electron. A state is therefore either occupied or empty, so that the number of electrons N_m in state m is either 0 or 1.

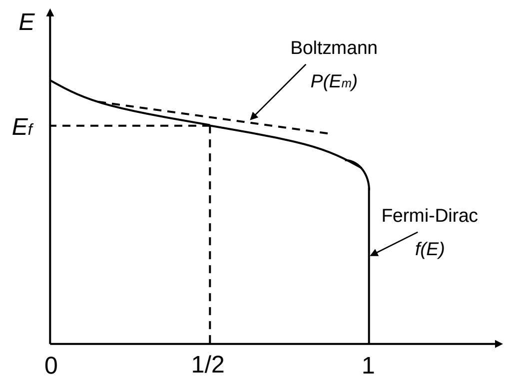
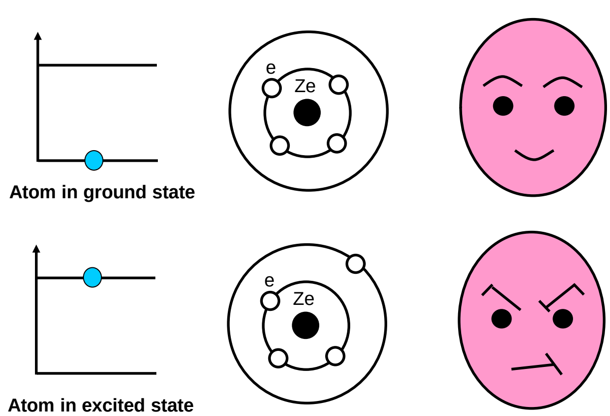


Figure 12.1-11 The Fermi-Dirac distribution *f(E)* is well approxiamated by the Boltzmann distribution *P(Em)* when *E>>Ef*.

$$f(E)=\frac{1}{\exp[(E-E_f)/k_BT)]+1}$$

3.Interactions of photons with atoms

- Semi-classical view of atom excitations



Spontaneous Emission

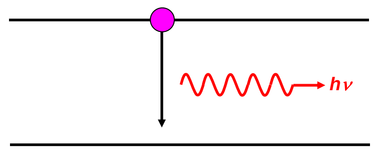


Figure 4.4-1 Spontaneous emission of a photon into the mode of frequency ν by an atomic transition from energy level 2 to level 1. The photon energy $h\nu=E_2-E_1$

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$$p_{sp} = \frac{c}{V}\sigma(\nu)$$ transition cross-section.

Decay of the number of the excited atoms

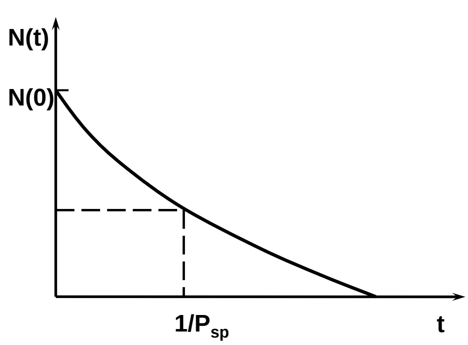

Figure 4.4-2 Spontaneous emission into a single mode causes the number of excited atoms to decrease exponentially with time constant 1/Psp

$$N(t) = N(0)\exp(-p_{sp}t)$$

Absorption

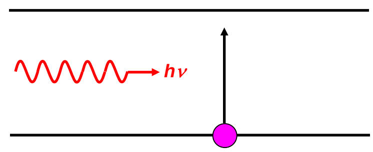

Figure 12.2-3 Absorption of a photon $h\nu$ leads to an upwoard transition of the atom from energy level 1 to energy level 2.

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$$P_{ab} = n\frac{c}{V}\sigma(\nu)$$

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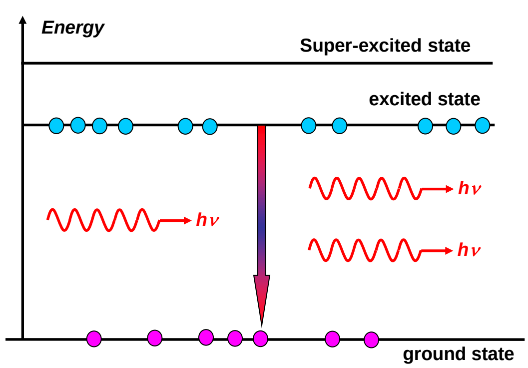
Stimulated Emission



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$$P_{st} = \frac{c}{V}\sigma(\nu)$$

$$P_{st} = n\frac{c}{V}\sigma(\nu)$$

$$W_i = P_{ab} = P_{st}$$

When a photon enters, it “knocks” an electron from the inverted population down to the ground state, thus creating a new photon. This amplification process is called stimulated emission.

Lineshape Function

Transition Strength

$$S = \int_{-\infty}^{\infty} \sigma(\nu) d\nu$$

Lineshape function

$$\sigma(\nu) = S g(\nu)$$

$$\int_{-\infty}^{\infty} g(\nu) d\nu = 1$$

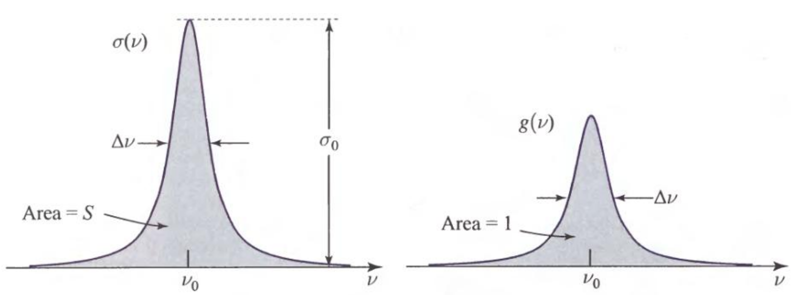

Figure The transition cross section σ(ν) and the lineshape function g(ν)

Total Spontaneous Emission into All Modes

shown in Sec. 9.1C, the density of modes for a three-dimensional cavity is $M(\nu) = 8\pi\nu^2/c^3$. This quantity approximates the number of modes (per unit volume of the cavity per unit bandwidth) that have the frequency ν; it increases in quadratic fashion.

The probability density of spontaneous emission into a single prescribed mode must therefore be weighted by the modal density. The overall spontaneous emission probability density is thus

$$P_{sp} = \int_0^\infty \left[\frac{c}{V}\sigma(\nu)\right][VM(\nu)]\,d\nu = c\int_0^\infty \sigma(\nu)M(\nu)\,d\nu.$$

Because the function $\sigma(\nu)$ is sharply peaked, it is narrow in comparison with the function $M(\nu)$. Since $\sigma(\nu)$ is centered about ν_0, $M(\nu)$ is essentially constant at $M(\nu_0)$,

$$P_{sp} = M(\nu_0)cS = \frac{8\pi S}{\lambda^2}$$

$$1/t_{sp} \cong P_{sp} = M(\nu_0)cS$$

$$P_{sp} = \frac{1}{t_{sp}}$$

$$S = \frac{\lambda^2}{8\pi t_{sp}}$$

- Relation between the Transition Cross Section and the Spontaneous Lifetime

$$\sigma(\nu) = \frac{\lambda^2}{8\pi t_{sp}} g(\nu)$$

$$\sigma_0 = \sigma(\nu_0) = \frac{\lambda^2}{8\pi t_{sp}} g(\nu_0)$$

- Transitions Induced by **Monochromatic** Light

$$\phi = \frac{I}{h\nu} \qquad n = \phi \frac{V}{c}$$

$$W_i = \phi\sigma(\nu)$$

- Transitions Induced by **Broadband** Light

$$W_i = \int_0^\infty \frac{\rho(\nu)V}{h\nu}[\frac{c}{V}\sigma(\nu)]d\nu$$

Since the radiation is broadband, the function $\varrho(\nu)$ varies slowly in comparison with the sharply peaked function $\sigma(\nu)$. We can therefore replace $\varrho(\nu)/\nu$ under the integral with $\varrho(\nu_0)/\nu_0$ to obtain

$$W_i = \frac{\rho(\nu_0)V}{h\nu_0} c\int_0^\infty \sigma(\nu)d\nu = \frac{\rho(\nu_0)}{h\nu_0} cS$$

$$W_i = \frac{\lambda^3}{8\pi h t_{sp}} \rho(\nu_0) \qquad (4.4\text{-}17)$$

Einstein Coefficients

Einstein did not have knowledge of (12.2-17). However, based on an analysis of the exchange of energy between atoms and radiation under conditions of thermal equilibrium, he was able to postulate certain expressions for the probability densities of the different kinds of transitions an atom may undergo when it interacts with broadband radiation of spectral energy density $\varrho(\nu)$. The expressions he obtained were as follows:

$$P_{sp} = A$$

$$W_i = B\rho(\nu_0)$$

$$P_{sp} = A$$

$$W_i = B\rho(\nu_0)$$

$$P_{sp} = \frac{1}{t_{sp}}$$

$$\Rightarrow$$

$$A = \frac{1}{t_{sp}}$$

$$B = \frac{\lambda^3}{8\pi h t_{sp}}$$

Homogeneous and inhomogeneous broadening

- To describe the distribution of the emitted intensity versus the frequency *v*, we define a **lineshape function** *g(v):*

$$\int_{-\infty}^{\infty} g(\nu)d\nu = 1$$

- →*g(v)dv* can be considered as a **priori probability** that a given spontaneous emission 2→1 will result in a photon whose frequency is between *v* and *v+dv*
- →Both the emission and the absorption are described by the same lineshape function *g(v)*
- →*g(v)* can be measured by measuring the profile of the absorption spectrum for the transition 1→2

Homogeneous Broadening

Radiated field $e(t) = E_0 e^{-t/\tau}\cos(\omega_0 t) = \frac{E_0}{2}[e^{i(\omega_0 + i\sigma/2)t} + e^{-i(\omega_0 - i\sigma/2)t}]$

$\sigma/2 = \tau^{-1}$ *Field decay rate*

Fourier Transform $E(\omega) = \int_0^\infty e(t)e^{-i\omega t}dt = \frac{E_0}{2}[\frac{i}{\omega_0 - \omega + i\sigma/2} - \frac{i}{\omega_0 + \omega - i\sigma/2}]$

At the vicinity of the resonant frequency w_0

$$\left|E(\omega)^2\right| \propto \frac{1}{(\omega_0 - \omega)^2 + (\sigma/2)^2}$$

Coresponded curves are called Lorentzian

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Lineshape Broadening

Because the lineshape function $g(\nu)$ plays an important role in atom–photon interactions, we devote this subsection to a brief discussion of its origins. The same lineshape function is applicable for spontaneous emission, absorption, and stimulated emission.

Life-time broadening

$$\Delta E = \Delta E_1 + \Delta E_2 = \frac{h}{2\pi}(\frac{1}{\tau_1} + \frac{1}{\tau_2}) = \frac{h}{2\pi}\frac{1}{\tau}$$

$$\Delta\nu = \frac{1}{2\pi}(\frac{1}{\tau_1} + \frac{1}{\tau_2})$$

Caused by the time uncertainty of the occupation of the energy level

$$g(\nu) = \frac{\Delta\nu/2\pi}{(\nu - \nu_0)^2 + (\Delta\nu/2)^2}$$

$$\sigma_0 = \frac{\lambda^2}{2\pi}\frac{1}{2\pi t_{sp}\Delta\nu}$$

$$\sigma_0 = \frac{\lambda^2}{2\pi}$$

Linewidth $\Delta\nu = \frac{\sigma}{2\pi} = \frac{1}{\pi\tau}$

Lineshape function $g(\nu) = \frac{\Delta\nu}{2\pi[(\nu-\nu_0)^2 + (\Delta\nu/2)^2]}$

$$\Delta\nu = \frac{1}{\pi}(\tau_u^{-1} + \tau_l^{-1} + \tau_{cu}^{-1} + \tau_{cl}^{-1})$$

Collision Broadening

Inelastic collisions, in which energy is exchanged, result in atomic transitions between energy levels. This contribution to the decay rates affects the lifetimes of all levels involved and hence the linewidth of the radiated field, as indicated above.

Elastic collisions, on the other hand, do not involve energy exchange. Rather, they cause random phase shifts of the wavefunction associated with the energy level, which in turn results in a random phase shift of the radiated field at each collision time.

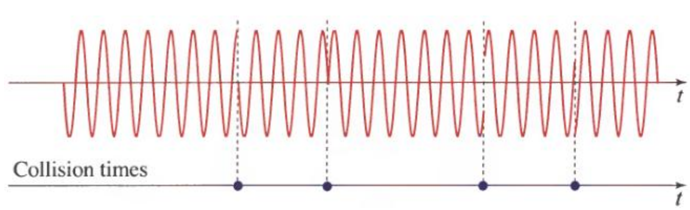


Figure 4.4-8 A sinewave interrupted at the rate f_{col} by random phase jumps has a Lorentzian spectrum of width $\triangle\nu = f_{col}/\pi$

Inhomogeneous Broadening

- Doppler effect

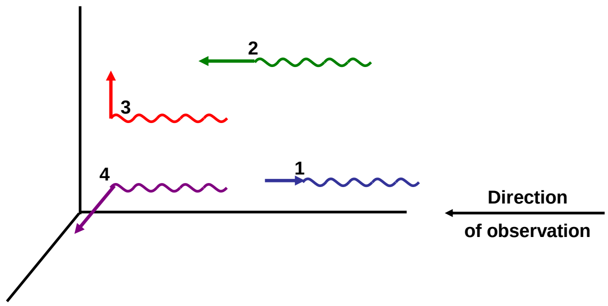


Figure 4.4-10 The radiated frequency is dependent on the direction of atomic motion relative to the direction of observation. Radiation from atom 1 has higher frequency than that from atoms 3 and 4. Radiation from atom 2 has lower frequency.

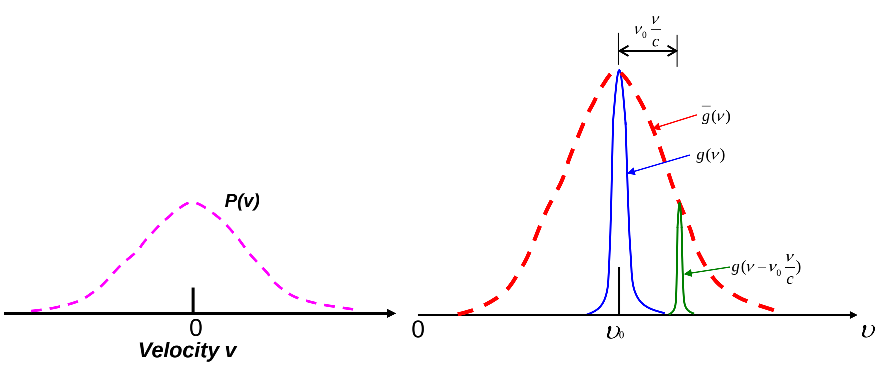


Figure 4.4-11 The velocity distribution and average lineshape function of a Doppler-broadened atomic system.

$$\bar{g}(\nu)=\int_{-\infty}^{+\infty} g(\nu-\nu_0\frac{V}{c})p(V)dV$$

Features of homogeneous broadening:

1. Each atom in the system has a common emitting spectrum width$\Delta\nu$. $g(\nu)$ describes the response of any of the atoms, which are indistinguishable
2. Due most often to the finite interaction lifetime of the absorbing and emitting atoms

Mechanisms of homogeneous broadening:

1. The spontaneous lifetime of the exited state
2. Collision of an atom embedded in a crystal with a phonon
3. Pressure broadening of atoms in a gas

Features of Inhomogeneous Broadening

1. Individual atoms are distinguishable, each having a slightly different frequency.
2. The observed spectrum of spontaneous emission reflects the spread in the individual transition frequencies (not the broadening due to the finite lifetime of the excited state).

Typical Examples:

- The energy levels of ions presents as impurities in a host crystal.
- *Random strain*
- *Crystal imperfection*

3. Thermal light

Thermal Equilibrium Between Photons and Atoms

$$P_{sp} = A \qquad A = \frac{1}{t_{sp}}$$

$$W_i = B\rho(\nu_0) \qquad W_i = P_{sp} = P_{st}$$

$$\frac{dN_2}{dt} = -\frac{N_2}{t_{sp}} + \frac{\bar{n}N_1}{t_{sp}} - \frac{\bar{n}N_2}{t_{sp}}$$

$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{k_B T}\right) = \exp\left(-\frac{h\nu}{k_B T}\right)$$

The average number of photons in a mode of frequency ν

$$\bar{n} = \frac{1}{\exp(h\nu / k_B T) - 1}$$

Black-body Radiation

$$\bar{E} = \frac{h\nu}{\exp(h\nu / k_B T) - 1}$$

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp(h\nu / k_B T) - 1}$$

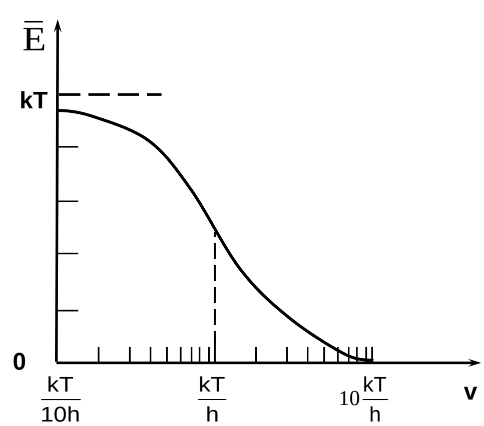
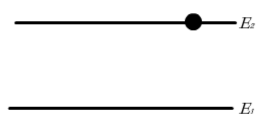


Figure 4.5-2 Semilogarithmic plot of the average energy E of an electromagnetic mode in thermal equilibrium at temperature T as a function of the mode frequency ν. At T=300K, $K_B T/h$ = 6.25THz, which corrsponds to a wavelength of 48um.



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Spontaneous emissionl

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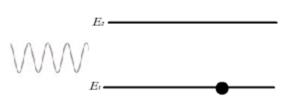

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Absorption

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Stimulated Emission

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