THERMAL SCIENCE
International Scientific Journal

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# ON THE GOODMAN HEAT-BALANCE INTEGRAL METHOD FOR STEFAN LIKE-PROBLEMS

ABSTRACT

Since the pioneering studies of Goodman on the application of the integral method to transient non-linear heat diffusion, much attention has been devoted nowadays to what is called heat balance integral method. The present paper considers this technique fifty years later. The genesis and earlier developments, when applied to Stefan like-problems, are reported hereafter. Its simplicity and efficiency are demonstrated. Some numerical results obtained using methods developed on the basis of the heat balance integral are compared. Furthermore, for problems including temperature profile behaviour, such as Stefan problem with forcing term (source or sink) this technique gives highly precise results and may, in some cases, lead to exact solutions.

KEYWORDS

heat balance integral, Stefan problems, analytical solution, source-sink term, phase-change, non-linear heat diffusion, moving boundary

PAPER SUBMITTED: 2008-04-04

PAPER REVISED: 2008-05-15

PAPER ACCEPTED: 2008-06-15

DOI REFERENCE: TSCI09020815

CITATION EXPORT: view in browser or download as text file

THERMAL SCIENCE YEAR 2009, VOLUME 13, ISSUE 2, PAGES [81 - 96]

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