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Technical Note

Application of dynamic vibration absorbers in structural vibration control under multi-frequency harmonic excitations

H.L. Sun *, P.Q. Zhang, H.B. Chen, K. Zhang, X.L. Gong

CAS Key Laboratory of Mechanical Behavior and Design of Materials, Department of Modern Mechanics, University of Science & Technology of China, Hefei, Anhui 230027, China

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Abstract

In this research, the response properties of a single-degree-of-freedom system under dual harmonic excitation are analyzed to provide some principles for the choice of time span and step in the simulations. The performances of dynamic vibration absorber (DVA) and state-switched absorber (SSA) are compared. The results indicate that dual DVAs almost have the same performance as the SSA. Moreover, dual DVAs compared with the SSA have some advantages such as lower ratio of tuning frequencies, more rapid optimization process and lower requirement for the anti-fatigue property of the material. Furthermore, the performances of different frequency-tuning methods are investigated. It is shown that the one–one method almost has the same performance as the optimization method and it does not need time-consuming optimization process.
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Keywords: Dynamic vibration absorber; State-switched absorber; Multi-frequency; Frequency-tuning

1. Introduction

Dynamic vibration absorbers (DVA) have been successfully used to attenuate the vibration of many structures. The DVA usually consists of a mass attached to the structure to be controlled through a spring-damper system. It is usually used to suppress a harmonic excitation at a given frequency (tonal tuning). The principal drawback of tonally tuned absorbers is its quite small effective bandwidth. Igusa analyzed the vibration control capabilities of multiple tuned mass dampers with natural frequencies distributed over a frequency range [1]. It is found that multiple tuned mass dampers can be more effective and robust than a single tuned mass damper with equal total mass.

In recent years, semi-active and active-passive vibration absorbers have been proposed to suppress harmonic excitations with time-varying frequency [2–18]. Semi-active vibration absorbers can be separated into several types: variable stiffness through mechanical mechanisms [2,8,16,17] or using controllable new materials [6,12,13,18], variable inductor connected in series with the piezoelectric patch for piezoelectric absorbers [7,9–11]. The first two types are used widely which can be tuned to the varying frequency by changing their resonant frequency. A lot of adaptive vibration absorbers with variable stiffness have been proposed and verified experimentally and shown that these devices can effectively suppress the vibration of the primary structure with wider-band frequency. Active-passive vibration absorbers by applying a control force on a passive vibration absorber can improve the performance. The active absorbers with different actuators and control laws have been investigated and implemented [3–5,14,15]. It has advantages such as large bandwidth, high vibration reduction level and fine robustness. But it needs large power requirement.

Most of researches focus on the applications of absorbers in the system under harmonic excitations with

* Corresponding author. Tel.: +86 551 3603724; fax: +86 551 3606459.
E-mail address: hlsun@ustc.edu (H.L. Sun).

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a single frequency. However, many systems in real applications are excited by multiple frequencies. For example a typical system is floating raft on which several rotary machines are mounted [19]. Many kinds of multiple-shunt piezoelectric shunt damping which could be made effective against several frequencies have been developed [11,14]. However, the frequencies were required to be known in advance such that a suitable filter network could be designed such that each shunt acted only against a single frequency, with all other frequencies blocked. The design and adaptation of the filter network are difficult and complex. Filipovic developed a band-pass active absorber applied to the vibration control of structure under multi-frequency harmonic excitations [5]. It can absorb vibration at all frequencies that belong to the band-pass range. However the main drawback of fully active approach is the requirement of large control force.

Cunefare et al. developed a kind of state-switched absorber (SSA) which is a single-degree-of-freedom oscillator, but one with a spring whose stiffness can assume a number of discrete values [20]. It can address the control of multi-frequency excitation. Holdhusen used magneto-rheological elastomers to fabricate an SSA for vibration control of continuous system [21]. The results showed that the SSA was effective over a larger bandwidth than that of a classical tuned vibration absorber. One of the drawbacks of SSA is that its switching control law is not easily expanded to the case of multiple frequencies excitations. In addition, for steady vibration under dual-frequency harmonic excitation, SSA still need change the stiffness frequently and the switch is more frequent as the spacing in excitation frequencies increased. This will decrease its working life.

The objective of the present research is to find a simple and effective method for vibration control of multi-frequency harmonic excitation by the comparisons of different DVAs and frequency-tuning methods. At first, the response properties of a single-degree-of-freedom system under dual harmonic excitation are analyzed. Secondly, the performances of a single DVA, two DVA and an SSA applied to a single-degree-of-freedom system are compared. The objective function to be optimized is the time averaged kinetic energy of the primary system. Lastly, the performances of different frequency-tuning methods are investigated.

2. Properties of response to multi-frequency harmonic excitation

The focus of this paper is on vibration reduction effects of different absorbers for multiple harmonic excitations. As such, our modeling and simulations will focus on simple, ideal lump-element models. A single-degree-of-freedom primary system under multi-frequency harmonic excitation is depicted in Fig. 1, where x, M_p, C_p, K_p are the displacement, mass, damping and stiffness of the primary system, respectively; F is the excitation force applied on the primary system; ω_1, ω_2, F_1, F_2 are the forcing frequencies

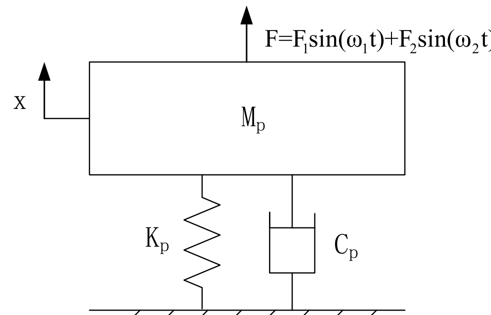

Fig. 1. Model of primary system.

and their corresponding amplitudes, respectively. Before absorbers are introduced to the primary system, its properties of response to multiple harmonic excitations are analyzed.

The equation of motion for the system in Fig. 1 can be written as

$$M_p\ddot{x}(t) + C_p\dot{x}(t) + K_px(t) = F_1\sin(\omega_1 t) + F_2\sin(\omega_2 t) \tag{1}$$

The driving point mobility of the primary system can be written as

$$H(\omega) = \frac{1}{-M_p\omega^2 + jC_p\omega + K_p} \tag{2}$$

where ω is the frequency of the harmonic excitation force.

According to the superposition property, the steady response of Eq. (1) can be written as

$$x(t) = |H(\omega_1)|F_1\sin(\omega_1 t + \phi_1) + |H(\omega_2)|F_2\sin(\omega_2 t + \phi_2) \tag{3}$$

where

$$\phi_1 = \arg(H(\omega_1)), \quad \phi_2 = \arg(H(\omega_2)).$$

Since the response includes two harmonic components, the vibration level of the primary system can not be measured simply by the amplitude of the response. So the root mean square of the response is analyzed. Eq. (3) can be discretized as

$$X(k) = |H(\omega_1)|F_1\sin(\omega_1 kT_{\mathrm{step}} + \phi_1) + |H(\omega_2)|F_2 \times \sin(\omega_2 kT_{\mathrm{step}} + \phi_2) \tag{4}$$

where T_{step} is the length of time step. The root mean square of the response is defined as

$$X_{\mathrm{RMS}} = \sqrt{\frac{X(1)^2 + X(2)^2 + \cdots + X(n)^2}{n}} \tag{5}$$

When the two forcing frequencies are close, beat phenomenon appears in the response. Therefore short period $T_s = 2\pi/(\omega_1 + \omega_2)$ and long period $T_l = 2\pi/(\omega_1 - \omega_2)$ are used to choose suitable time step T_{step} and time span T_{span}. Fig. 2 shows how the root mean square of the response changes with the variations of the time step and span.

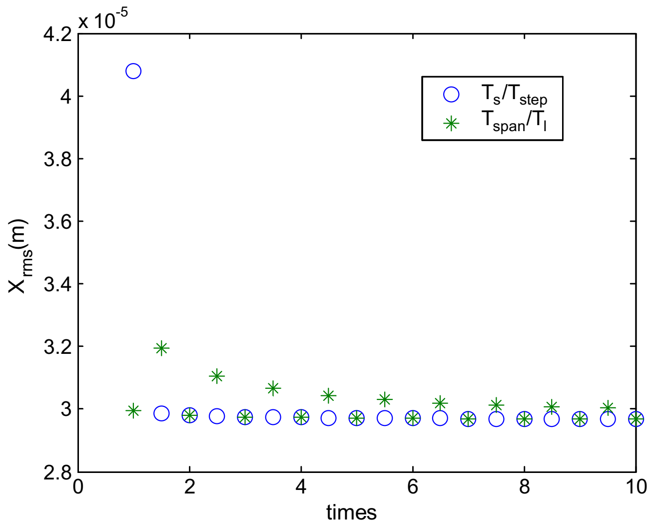

Fig. 2. Root mean square of the response.

From Fig. 2, the root mean square of the response approaches a constant when the time step is short enough or the time span is long enough. So it can be used to measure the vibration level. On the other hand, when time span is integer times of long period, the small time span can make the root mean square of the response approach a constant. And further when $T_s/T_{step} \geqslant 2$ and $T_{span}/T_l \geqslant 5$, the truncation errors are less than 0.4% and 2.7%, respectively.

3. Comparison of a single DVA, a single SSA and two DVAs

The comparison described below is intended to demonstrate the level of vibration control achieved by a single DVA, a single SSA and dual DVAs. Fig. 3 depicts a single DVA, a single SSA and dual DVAs, each mounted on identical primary systems which have the same parameters as model in Fig. 1. x_a, x_{ssa}, x_1, x_2 are the displacement of the single DVA, single SSA and dual DVAs, respectively; M_a, M_{ssa}, M_1, M_2 are the corresponding mass; K_a, K_{ssa}, K_1, K_2 are the corresponding stiffness; C_a, C_{ssa}, C_1, C_2 are the corresponding damping. It is assumed that identical forces with two harmonic components are applied to the primary mass of each system.

3.1. Equations of motion

Following the work of Cunefare et al. [20], the dynamical systems described above may all be cast in terms of a state-space representation as

$$\dot{X}(t) = A_{\text{on-line}}X(t) + Bu(t) \tag{6}$$

where $A_{\text{on-line}}$ is the only term that depends upon the stiffness state of the system. In Eq. (6),

$$X(t) = \begin{Bmatrix} q(t) \\ \dot{q}(t) \end{Bmatrix} \tag{7}$$

is the state-vector. $q(t)$ is $[x, x_a]'$ for single DVA, $[x, x_{ssa}]'$ for SSA and $[x, x_1, x_2]'$ for two DVAs where the superscript sign ($'$) denotes transpose of matrix.

The coefficient matrices of two system equations with a single DVA and SSA have the same form as

$$A_{\text{on-line}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_p+K}{M_p} & \frac{K}{M_p} & -\frac{C_p+C}{M_p} & \frac{C}{M_p} \\ \frac{K}{M} & -\frac{K}{M} & \frac{C}{M} & -\frac{C}{M} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M_p} \\ 0 \end{bmatrix} \tag{8}$$

where K, M are the stiffness and mass of the single DVA and SSA, respectively. For a two-state SSA, the switching control law developed by Cunefare et al. is used to adjust its stiffness [20]. The switching principle is

$$K_{ssa}^{next} = \begin{cases} K_{ssa1} & \text{if } \dot{x}(\dot{x}_{ssa} - \dot{x}) > 0 \\ K_{ssa2} & \text{if } \dot{x}(\dot{x}_{ssa} - \dot{x}) < 0 \\ K_{ssa} & \text{if } \dot{x}(\dot{x}_{ssa} - \dot{x}) = 0 \end{cases} \tag{9}$$

where K_{ssa}^{next} is the next value of the SSA stiffness K_{ssa}.

The coefficient matrices of the system with dual DVAs can be written as

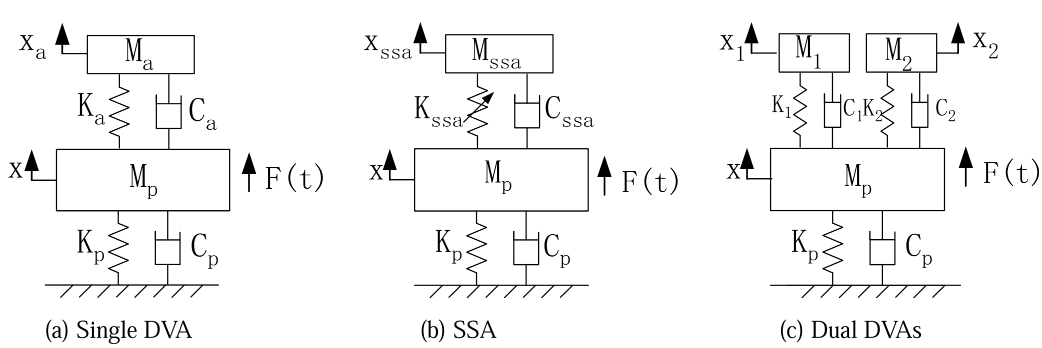

Fig. 3. Single DVA, SSA and dual DVAs mounted on identical primary systems.

$$A_{\text{on-line}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{K_p+K_1+K_2}{M_p} & \frac{K_1}{M_p} & \frac{K_2}{M_p} & -\frac{C_p+C_1+C_2}{M_p} & \frac{C_1}{M_p} & \frac{C2}{M_p} \\ \frac{K_1}{M_1} & -\frac{K_1}{M_1} & 0 & \frac{C_1}{M_1} & -\frac{C_1}{M_1} & 0 \\ \frac{K_2}{M_2} & 0 & -\frac{K_2}{M_2} & \frac{C_2}{M_2} & 0 & -\frac{C_2}{M_2} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M_p} \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

where K_1, K_2 are the stiffness of dual DVAs; C_1, C_2 are the damping of dual DVAs.

The iterative solution of Eq. (6) for the system response is

$$X(k+1) = A_{di}X(k) + B_{di}u(k) \tag{11}$$

and

$$A_{di} = e^{A_{\text{on-line}}T_{\text{step}}}, \quad B_{di} = A^{-1}_{\text{on-line}}(A_{di} - I)B \tag{12}$$

where T_{step} is the time step of the discrete simulation.

As a basis for the three methods, the DVA mass, SSA mass and the total mass of dual DVAs are selected to be equal and each is 10% of the primary system mass. The excitation and resonance frequencies are nondimensionalized by the natural frequency of the primary system, $\omega_0 = (K_p/M_p)^{1/2}$. Both the single DVA and SSA are presumed to have damping elements such that their damping ratio at the frequency ω_0 is 5%. The damping elements of dual DVAs are identical and their damping is half of the single DVA.

Similar to the work [20], the performances of a single DVA, SSA and dual DVAs optimized for the identical set of exciting frequencies ω_1 and ω_2 are compared. Direct-search optimization method is used to determine the optimal DVA, SSA and dual DVAs stiffness for each forcing frequencies ω_1 and ω_2. In direct-search optimization, all values of tuning frequency within a defined design space are tested through a time simulation for each test. The optimization tuning frequency is defined as the one that yields the minimum average kinetic energy of the primary mass at the end of the simulation. For the single DVA, this optimization process requires a search over only one tuning frequency; the SSA and two DVAs require a search over combinations of pairs of tuning frequencies. According to the optimization tuning frequency, the DVA, SSA and two DVAs stiffness K_a, K_{ssa1}, K_{ssa2}, K_1 and K_2 are determined.

3.2. Comparison of vibration reduction performance

During the simulation, the forcing frequency ω_1/ω_0 is fixed and ω_2/ω_0 varies between ratios of 0.9–1.1, in steps of 0.02. The design space for the optimization tuning frequency is defined as spanning 0.6–1.2 in steps of 0.04. The comparison of three methods is shown in Fig. 4. The smaller average kinetic energy means the lower vibration level of the primary mass, that is, the corresponding absorber has better performance. When the spacing of the two forcing frequencies is small, the performances of three methods are almost identical. Both SSA and dual DVAs have better performances than single DVA as the spacing of the two forcing frequencies increases. In addition, dual DVAs almost have the same performance as the SSA for all combinations of ω_1 and ω_2.

3.3. Comparison of tuning frequency

Fig. 5 indicates the ratio between the upper and lower tuning frequencies of the SSA and dual DVAs in order to achieve the performance shown in Fig. 4. The ratio of two tuning frequencies of SSA fluctuates more acutely

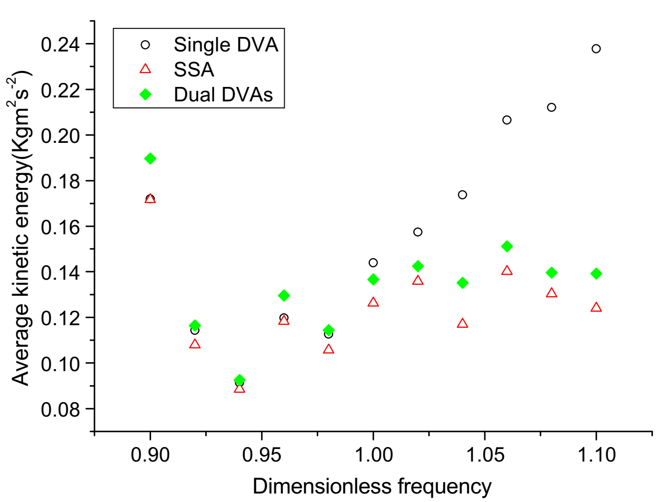

Fig. 4. Performance comparison of a single DVA, SSA and dual DVAs.

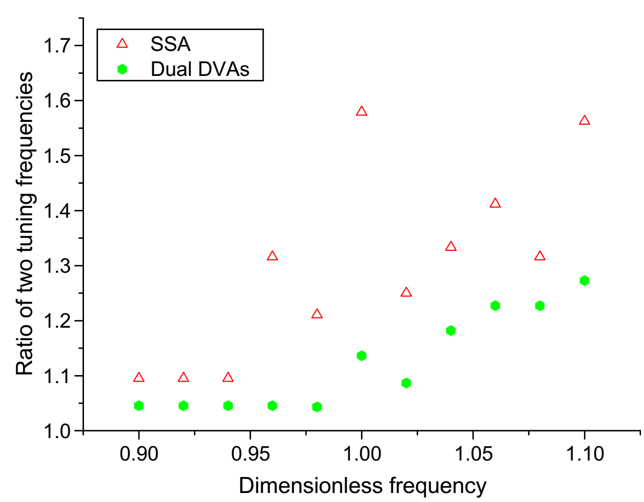

Fig. 5. Ratio of two tuning frequencies in SSA and dual DVAs.

and is much larger than that of dual DVAs. The larger fluctuation indicates that the tuning frequencies of SSA are more sensitive to the forcing frequencies. For an adaptive absorber tuned by adjusting stiffness, the larger ratio means that the properties of smart material used by absorber should be able to vary in broader range. Therefore, the method using dual DVAs can be implemented more conveniently. However, the convenience is at the cost of the increase of number of DVA. On the other hand, multiple absorbers are used at the same time in many applications [1,22]. For these applications, the number of DVA can not become the limitation of dual DVAs.

3.4. Tuning-to-forcing frequency ratio of dual DVAs

Fig. 6 shows the ratio between the two tuning frequencies of dual DVAs and the two forcing frequencies. ω_{n1} and ω_{n2} are the lower and upper tuning frequencies, respectively. It is shown that both ratios are nearly equal to unity. This means that each of two DVAs, respectively suppresses the vibration excited by one of the two forcing frequencies as traditional absorbers for single harmonic excitation. In addition, the ratios fluctuate near unity specially when the dimensionless frequency is unity. One of its reasons is that the optimization process is based on discrete frequencies and the tuning frequencies may be optimal or sub-optimal. Moreover for a single harmonic excitation, the performance of absorber depends on the spacing between its natural frequency and excitation frequency but whether the natural frequency of absorber is larger or less than the excitation frequency, which can lead to the fluctuating in Figs. 5 and 6.

On the other hand, from Eqs. (8)–(12), the coefficient matrices of Eq. (12) for a SSA vary at every switching point, that is, the optimization process of the tuning frequency of SSA involves a lot of inverse matrix computations that are time-consuming. However, the coefficient

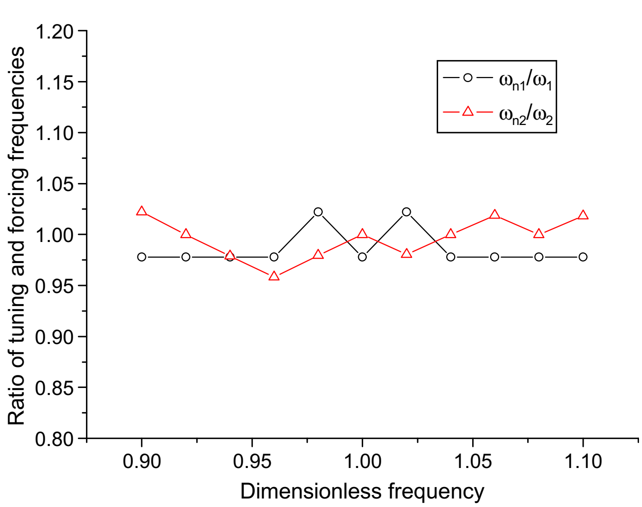

Fig. 6. Tuning-to-forcing frequency ratios of dual DVAs.

matrices of Eq. (12) for dual DVAs are constant for every forcing frequency combination.

4. Frequency-tuning method

In the above analysis, the optimization process of dual DVAs based on the minimum average kinetic energy of primary system needs to test all combinations of two forcing frequencies. If the excitation has more components, the optimization process still needs a great deal of time. On the other hand, when the primary system is a continuous structure, the measurement of its average kinetic energy requires a lot of sensors distributed on the primary system. So it is valuable to find a simpler frequency-tuning method. Traditional absorber is generally used to suppress the vibration excited by single harmonic force. The absorber should be designed such that its natural frequency is tuned to the excitation frequency.

For the excitation with multiple frequencies, according to Fig. 6, the optimal tuning frequencies are almost equal to the forcing frequencies. Therefore, one–one method is proposed to tune the DVAs to suppress the vibration under multi-frequency harmonic excitation. The method requires that the number of the tuning frequencies of absorbers equals that of forcing frequency components and their relation is one–one, that is, each of the tuning frequencies is equally matched to one of the forcing frequencies. If every absorber has only a tuning frequency, the proposed method needs the same number of absorbers as forcing frequency components. The method can be considered as the extension for the frequency-tuning method of traditional absorber used to suppress the vibration excited by single harmonic force.

To assess the relative performance of the optimized dual DVAs depicted in previous part and the dual DVAs tuned by one–one method, the logarithm of the ratio of average kinetic energies (AKE) is used as a figure

$$\beta = 10\lg\left(\frac{\text{AKE of the optimization method}}{\text{AKE of the one–one method}}\right) \tag{13}$$

When Eq. (13) yields negative value, the optimized dual DVAs outperformed the dual DVAs tuned by one–one method, i.e., the former AKE is lower than the latter.

4.1. Effect of frequency step on the relative performance

During the simulation, the forcing frequency ω_1/ω_0 is fixed and ω_2/ω_0 varies between ratios of 0.9–1.1, in steps of 0.02. The design space for the optimization tuning frequency is defined as spanning 0.6–1.2 with three kinds of frequency steps 0.04, 0.02 and 0.01. The damping ratio of the DVAs at the frequency ω_0 is 5%. Fig. 7 compares the performances of the optimized two DVAs and the two DVAs tuned by one–one method. The results show that when the frequency step is larger, the optimization method has lower performance than one–one method. As the frequency step is shortened, the two methods almost have

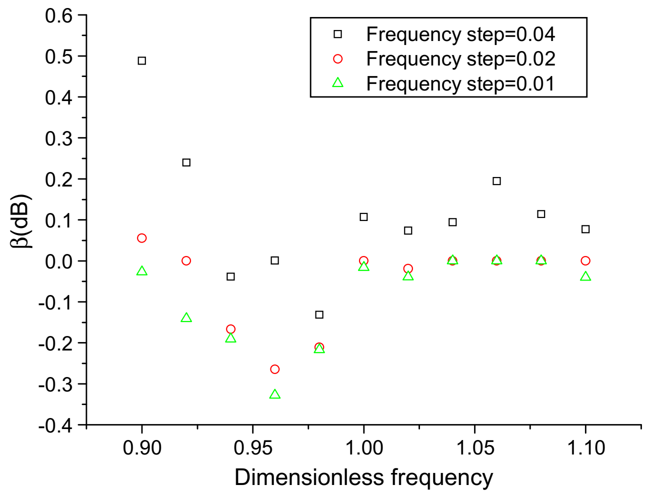

Fig. 7. Performance comparison as defined by Eq. (13) under different frequency steps.

the same capability. Therefore, in order to get good performance, the optimization method should have short enough frequency step.

4.2. Effect of damping ratio on the relative performance

Fig. 8 presents the effect of damping ratio of absorbers on the relative performance between the optimization and one–one method. The frequency step for the optimization tuning frequency in Fig. 8 is fixed on 0.01. The optimization method outperforms slightly one–one method when the spacing of the two forcing frequencies is smaller. On the whole, the performances of the two methods only have a little difference and the difference decreases as the damping ratio reduces. It is similar with the result of traditional absorber used to suppress the vibration excited by single harmonic force. That is, the smaller the damping ratio is, the closer the optimal tuning frequency is to the forcing frequency.

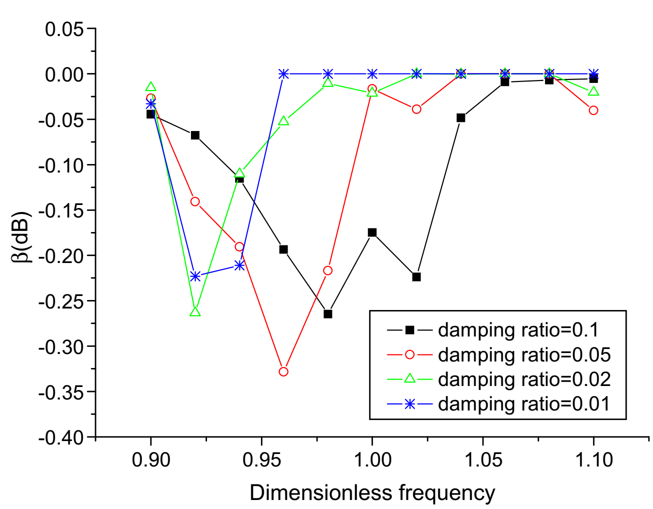

Fig. 8. Performance comparison as defined by Eq. (13) under different damping ratios.

From the above analysis, the one–one method almost has identical performance with the optimization method and it does not need time-consuming optimization process. In addition, the method can be easily expanded to the case of the excitation including more frequency components. Moreover, if the DVA can adjust its natural frequency adaptively, the method is also suitable for multiple time-varying frequencies harmonic excitation. Therefore, if the requirement for the vibration reduction effect is not strict, the one–one method is a more convenient and practical method for engineering applications.

5. Conclusions

The present research explores the application of DVA to the vibration control of multiple frequency harmonic excitations. The response properties of a single-degree-of-freedom system under dual harmonic excitation are analyzed to provide some principles for the choice of time span and step in the simulations. The performances of a single DVA, a single SSA and dual DVAs are compared. The results indicate that dual DVAs almost have the same performance as the SSA and they both have better performances than single DVA. In addition, two DVAs compared with the SSA have some advantages such as lower ratio of tuning frequencies, more rapid optimization process and lower requirement for the anti-fatigue property of the material.

Furthermore, the performances of different frequency-tuning methods for excitation with multiple frequency components are investigated. The results show that the one–one method almost has identical performance with the optimization method and it does not need time-consuming optimization process. In addition, the method can be easily expanded to the case of the excitation including more frequency components. However, the method requires that the number of the tuning frequencies of absorbers equals that of forcing frequency components.

On the whole, if the material used for variable stiffness element has high enough performances, the SSA is a good choice for dual harmonic excitation. Otherwise, if the excitation has three or more frequency components and the requirement for the number of absorbers is flexible, the one–one method is a more convenient and practical alternative for engineering applications.

Acknowledgement

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