# STUDY ON THE METHODS OF SUPER-RESOLUTION IMAGE RECONSTRUCTION

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#### **ABSTRACT:**

With the rapid development of space technology and its related technologies, more and more remote sensing platforms are sent to outer space to survey our earth. Recognizing and positioning all these space objects is the basis of knowing about the space, but there are no other effective methods in space target recognition except orbit and radio signal recognition. Super-resolution image reconstruction, which is based on the image of space objects, provides an effective way of solving this problem. In this paper, the principle of super-resolution image reconstruction and several typical reconstruction methods were introduced. By comparison, Nonparametric Finite Support Restoration Techniques were analyzed in details. At last, several aspects of super-resolution image reconstruction that should be studied further more were put forward.

#### 1. INTRODUCTION

As the space technology develops faster and faster, we already have many platforms flying above our earth. Recognizing and positioning these space objects is often before knowing the earth. Additionally, as human beings explore the space and realize the danger of planetoid, we need to know more about the outer space, not only for our curiosity but also for our safety. Super-resolution image reconstruction is a new effective method to detect all the space objects. And through this technology, we can generate images that are near or even surpass diffraction limit, which can help a lot in space objects recognition. On the other hand, super-resolution images are inherently identical to remote sensing images, so some of the technologies in super-resolution image processing are also useful in remote sensing image processing.

In 1991, B.R.Hunt applied this method to astronomical image reconstruction and put forward PMAP algorithm, which is based on maximum Poisson posterior estimate. 1995, B.R.Hunt pointed out that reason we can reconstruct super resolution image is there are high frequency information in low frequency components. Recent years, more and more researchers focus their studies on super-resolution image reconstruction and gain satisfying results in practice. In this paper, the principle of super-resolution image reconstruction is introduced and commonly used methods in super-resolution reconstruction are analyzed. As a research hotspot, Nonparametric Finite Support Restoration Techniques are studied in details.

# 2. PRINCIPLE OF SUPER-RESOLUTION IMAGE RECONSTRUCTION

# 2.1 Definition of Super Resolution

Noncoherent transfer function of an optical system is the autocorrelation of its pupil function, which means that the transfer function is necessarily band-limited. In another way, the value of transfer function should be zero when frequency determined by diffraction limit is above certain value. Apparently, deconvolution can only restore the spectrum of object to diffraction limit and cannot surpass it. However, by

using Fourier transformation, we can get resolution above diffraction limit in theory. The restoration technology which is trying to restore the information above diffraction limit is called Super-resolution techniques. And the methods used in these techniques can be called Extrapolation of Band-limited.

Diffraction limiting Images of space objects can be obtain through high-resolution restoration of speckle images of these objects. But, with super-resolution information, resolution can be improved further by restoration and reconstruction of neardiffraction-limit images.

# 2.2 Principle

Super-resolution image reconstruction is based on the theory of Analytic Continuation, which means reconstruction of the whole analytic function according to its values in certain area. Because of diffraction of lights, spectrum distribution of certain image is infinite in space and optical system truncates its frequency to obtain frequency-truncated image that is finite in space. Generally, truncation function cannot be band-limited, but a diffraction limited optical system's truncation is band-limited, therefore, the reconstruction of whole spectrum function or just spectrum function above certain frequency is possible.

Assume the imaging model:

$$g(x, y) = h(x, y) * f(x, y) + n(x, y)$$
(1)

Where, h(x, y) is the point spread function (PSF), f(x, y) is ideal image, g(x, y) is the original image and n(x, y) is the noise.

Its Fourier transformation is:

$$G(u,v) = H(u,v) \cdot F(u,v) + N(u,v) \tag{2}$$

Super-resolution reconstruction is to perform analytic continuation to F(u,v) to extend its support domain by using prior information of objects and posterior processing technologies, and then get a new PSF  $H^{'}(u,v)$ . H(u,v) also has the extended support domain, thus the resolution of image is improved.

#### 2.3 Classification of Methods

To the speckle image that is affected by the aerosphere disturbing, there are a few methods to improve the resolution of image. According to the processing domain, the methods can be divided into space domain method and frequency domain method. According to whether the PSF is known, there are Deconvolution and Blind Deconvolution methods, among which Blind Deconvolution is reconstructing images by estimating PSF.

There is usually little prior information in images and the PSF of image is hard to obtain, so blind deconvolution is oftener used to reconstruct the image, and people study more on this method.

According to whether restoration of PSF is combined with the image restoration, typical blind deconvolution methods can be of two kinds:

- (1) The recognition of PSF is separated with the restoration of image. PSF is obtained first, and traditional restoration methods are used to compute the estimate of original image.
- (2) The recognition of PSF and restoration of image are performed at the same time, so this kind of method is more complicated.

### 3. TYPICAL RECONSTRUCTION METHODS

There a few methods widely used in blind deconvolution, including Priori Blur Identification Method, Zero Sheet Separation, Auto Regressive Moving Average (ARMA) Estimate and Nonparametric Finite Support Restoration Techniques (NFSRT). This section will describe these methods in general.

### 3.1 Priori Blur Identification Methods

Priori Blur Identification Methods restore the image by recognizing PSF before restoring image. When using these methods to restore image, we usually assume that PSF is symmetrical, and the parametrical model of PSF is given. Widely used parametrical models include motion blurring caused by relative linear motion of camera and defocus-blurring of camera.

Based on this assumption, people put forward methods that make use of some features of blurred or original image to estimate PSF. These features can be special point and line in image or other features. Once PSF is determined, typical image restoring methods can be adopted to estimate original image. Priori blur identification is widely used in image restoration because of its simplicity, but its main restraint is that the parametrical model of PSF must be given while in many situations, it is not available. Additionally, in astronomical or X ray images, PSF is usually of Gauss form which makes the zero point of PSF do not exist, so this method is not applicable.

#### 3.2 Zero Sheet Separation

Zero Sheet Separation was put forward by Lane and Bates (1987). Its principle can be described as follow.

In multi-dimension z-transfer, zero points of z-transfer of a k dimensional sequence are almost sequential and positions on a (2k-2)dimensional hyper-surface. Assume that there are r convolutions of multi-dimensional sequence f1, f2,....,fr, and its Z-transformation is:  $F1 \times F2 \times F3... \times Fr$ .

If the hyper-surface which each zero point of Fi is on can be separated from each other, we can obtain each efi, where c is ratio factor. That is the concept of zero sheet separation.

Under the concept of zero sheet separation, the restoration of 2D image is translated into factorization of 2D polynomial. It is intuitive, but when bringing it into practice, serious problems appear. The main problem is it is very difficult to associate, cluster and trace all the roots of the polynomial and the roots are sensitive to the noise. So no real practical algorithm has been put forward.

# 3.3 Auto Regressive Moving Average (ARMA) Estimate.

This method regards the original image as a 2 dimensional AutoRegressive (AR) process and PSF model as a 2 dimensional Moving Average (MA). So the blurred image can be described as a noised observation of AutoRegressive Moving Average (ARMA). Therefore, blind deconvolution is translated into the problem of determining the parameter of ARMA.

There are several algorithms, including Maximum Likelihood, Generalized Cross Validation (GCV), Neural Network and High Order Statistics (HOS) and so on. They all have good robustness on noise, but when there are too many parameters, they cannot convergent to global optimality.

# 3.4 Nonparametric Finite Support Restoration Techniques (NFSRT)

Nonparametric Finite Support Restoration Techniques don't need to establish the parametric model of original or blurred image, and there are not too many strict restraints, so they are widely used in image restoration. These useful techniques are studied by more researchers, and we will discuss them in details in next section.

# 4. NONPARAMETRIC FINITE SUPPORT RESTORATION

In the degraded model of real image, as presented by (1), the existed linear image restoration algorithms all assume that the PSF is given, and try to obtain its inverse and make use of a lot of information about PSF, real image and noise to decrease noise. However, PSF is often unknown to us, and we don't have much information about the original image. For this problem, researchers put forward methods that restore image and obtain the PSF at the same time.

Different from the priori blur identification methods, in nonparametric finite support restoration techniques, the parametric model of original or blurred image is not necessary to assume. There are several methods need to discuss, including Iterative Blind Deconvolution, Richard-Lucy algorithm,

Nonnegativity And Support constraints Recursive Inverse Filtering (NAS-RIF) and so on. These methods add necessary restraints to the optimized standards and assume that original image is non-negative and the objects have finite support domain and the background is pure black or white.

#### 4.1 IBD (Iterative Blind Deconvolution)

Iterative Blind Deconvolution (IBD) was put forward by Ayers and Dainty (1988). It is one the most commonly used methods in blind deconvolution. Based on Fourier Transformation, this method causes less computation and has good anti-noise capability. The main drawback is that convergence of the iterative process is not guaranteed. Besides, the original image can have great impact on the final result.

The algorithm can be described as follow:

Figure 1 The process of IBD

 $\hat{f}(x,y)$  is the estimate of original image, its Fourier transformation is  $\hat{F}(u,v)$ ,  $\hat{h}(x,y)$  is the estimate of PSF, and its Fourier transformation is  $\hat{H}(u,v)$ . After a random initial value of image is given, the iterative process starts, where image domain restraints and frequency restraints are added in. These restraints are about the information of image and PSF.

In IBD, Wiener filter is adopted to estimate the image in frequency domain, so IBD is not sensitive to noise. But because its convergence is related with initial value of image, the convergence and the uniqueness of solution cannot be guaranteed.

Here are 3 experimental results on an image which are made through different iterations.







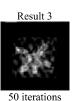


Figure 2 Experiments on IBD

From the figure2, we can see that the number of iteration that a better result requires is uncertain.

# 4.2 Richard-Lucy Algorithm

The main deficiency of many blind deconvolution algorithms is to process the priori information f(x, y) or h(x, y) directly, to conquer this, in 1970s, Richard-Lucy adaptive iterative algorithm.

The iterative process can be described as follow:

$$f_{k+1}(x,y) = f_k(x,y) \cdot (h(x,y) * \frac{g(x,y)}{r_k(x,y)})$$
(3)

Where, \* is the relevant operator,  $r_k(x, y)$  presents the reblurred image:

$$r_k(x, y) = f_k(x, y) * h(x, y)$$
 (4)

This method can increase the anti-noise capability to Poisson noise. If the noise is of Gauss form, the iterative can be as follow:

$$f_{k+1}(x,y) = f_k(x,y) + h(x,y) * (g(x,y) - r_k(x,y))$$
 (5)

The best iterative scheme has not been put forward yet, because the iteration of PSF and the iteration of image are not balanced. The follow experimental results show the effect of RL:



Figure 3 The experimental results on RL

The 2 results were obtained by using PSFs with size of13\*13 and 11\*11 through 12 iterations. We can see that the outer ring became clearer and the texture shows up.

#### 4.3 NAS-RIF

The aim of blind deconvolution is to reconstruct a reliable estimated image from a blurred image. D.Kundur put forward NAS-RIF algorithm (Nonnegative and Support Constrants Recursive Inverse Filtering) to achieve this aim.

In NAS-RIF, our aim is that, based on given image g(x,y), make a estimation  $\hat{f}(x,y)$  of target image. The estimation can be obtained by minimizing a error function which contains the support domain of image and nonnegative information of pixels. The solution that makes the error function globally optimized is called feasible solution. In theory, the estimation  $\hat{f}(x,y)$  is equivalent to the real image f(x,y), but they are different in ratio and position offset, which can described as follow:

$$\hat{f}(x, y) = Kf(x - D_x, y - D_y)$$
 (5)

Here is the flow of NAS-RIF: Firstly, put the blurred image g(x, y) into a 2 dimensional changeable FIR

filter u(x,y), whose output is an estimation  $\hat{f}(x,y)$  of real image. And then, through a non-linear filter, the estimation is projected into a real image space whose characteristic is known by using a non-expansive mapping. The difference between projected images  $\hat{f}_{NL}(x,y)$  and  $\hat{f}(x,y)$  is used as an error signal e(x,y) to update filter u(x,y). Through iterations, image can be restored. Figure 4 describes the flow.

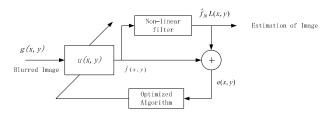


Figure 4 The flow of NAS-RIF

The greatest advantage of this algorithm is we don't need to know about the priori information of original image and the parameters of PSF, all we have to do is to determine support domain of target area and to make sure the estimation of image is nonnegative. Another advantage is that this algorithm contains a process that minimizes the cost function of convex set, which makes sure the function can convergent to global least. The disadvantage of NAS-RIF is that it is sensitive to noise, so it is only proper for images with symmetrical background.

Here are the results of experiments on two astronomic images.

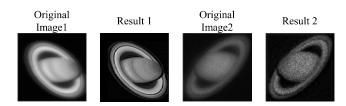


Figure 5 The experimental results on NAS-RIF

In processing of original image1, the size of PSF is 23\*23, the number of iteration is 7. We can see that, comparing with original image, textures of image become richer and image becomes clearer. In processing of original image2, the size of PSF is 21\*21, and the number of iteration is 5. From the result, it can be seen that, for the image with noise, the reconstruction effect of NAS-RIF is not so good, which proves that it is sensitive to noise.

# 4.4 Comparison

The several blind deconvolution algorithms that have been discussed above are being studied deeply recent years, and they all have good results. They have their own characteristics and can be used according to different situations. In this section, we will compare them in 3aspects: the quality of restored image, restoring speed and anti-noise capability.

(1)Quality of restored image.

In these 3 algorithms, IBD has the worst restoring quality. It can only obtain estimation near sample, so it is proper to be treated as initial value of image. For severe blurred image, NAS-RIF can have a satisfying restoring effect and the quality of restored image is better.

#### (2)Restoring speed.

Speed can be compared from the algorithm's complexity and convergence. IBD is the simplest and easy to implement, but its problem is that it has no convergence. So sometimes, there should be some restraints added in and it requires more iterations. NAS-RIF is more complex, but it has the best convergence.

# (3)Anti-noise capability

Because IBD uses the Wiener filter, so it is not sensitive to noise, even if the signal-to-noise ratio is low, the noise doesn't has much impact on restoration quality. NAS-RIF has the worst anti-noise capability.

#### 5. CONCLUSION

The researches on super-resolution image reconstruction mainly only consider the situation that degraded model is linear and noise is neglected and systemic analyzing method and filter designing method have not been formed yet. The further researches may have following aspects:

(1) The improving of existing algorithms and research on new algorithms

From the discussion of this paper, we can see that the existing algorithms have their own disadvantages which need to improve. For example, for IBD, how to choose initial values to guarantee the convergence; when stop the iteration to obtain the best restoration effect. For NAS-RIF, how to make it less sensitive to noise and how to extend it to images with non-symmetrical background are worth studying. Putting forward new methods to solve the image blind deconvolution problems is also the hotspot in further study.

#### (2) New algorithms based on non-linear degraded model

In practice, strictly speaking, all degraded models are non-linear. For simplicity, we often replace the non-linear model with linear model, but when the image model is severely non-linear, effect would be not satisfying. So studying algorithms based on non-linear model is also one of the developmental trends.

# (3) Study on denoising algorithms

The existence of additive noise makes the restoration of image an ill-conditioned problem, and because common assumptions only involve statistical features of noise, it is impossible to remove the noise from the degraded image completely. Besides, the existence of noise makes the restoration effect worse. Recently, most algorithms regard noise as Gauss form, but there also are other kinds of noises, using one kind of noise model is not enough for all the situations. Therefore, study on denoising algorithms is also meaningful to image reconstruction.

#### (4) Real-time algorithms

The complexity of algorithm is an important obstacle in its application. How to improve the processing speed is very important for the application of algorithm.

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