

# 一类非自治两捕食者-两互惠食饵系统的持久性和稳定性

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**摘要:** 研究了捕食者均具有 Beddington-DeAngelis 功能性反应的两捕食者-两互惠食饵系统. 利用比较原理论讨论了系统的一致持久性, 并通过构造 Liapunov 函数, 给出了系统全局渐进稳定的充分条件. 此外, 当系统的各系数为周期性时, 得到了正周期解存在唯一且全局渐进稳定的充分条件.

**关键词:** 互惠性; 捕食系统; 功能反应; 持久性; 稳定性

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种群动力学是生物数学理论中的一个重要分支, 研究种群动力学对渔业生产中控制种群数量、指导收获等方面有着重要的意义. 种群之间基本关系有捕食与被捕食、竞争和互惠共生三种. 已有大量文献研究了种群间的捕食与被捕食关系<sup>[1-3]</sup>, 如文献[1]研究的是食饵具有扩散效应的捕食系统, 并给出了系统持久共生的充要条件. 对于互惠系统的研究也较为详尽<sup>[4-6]</sup>, 如文献[4]讨论了一个具有阶段结构的非自治互惠系统, 得到了系统持续生存的充分条件和有关周期解的若干结论. 但实际上, 许多物种之间的关系并非是单一的, 而是相对复杂一些<sup>[7-9]</sup>. 例如文献[7]研究的是一类两互惠捕食者-食饵系统的动力学行为, 证明了系统的一致持久性, 并获得了正周期解的存在唯一性和全局吸引性. 笔者在此主要研究两个捕食者去捕食

两个具有互惠关系食饵的模型, 且捕食者对食饵均具有 Beddington-DeAngelis 功能性反应.

## 1 模型及相关介绍

### 1.1 模型简介

笔者研究的模型系统如(1)式所示.

在系统(1)中,  $x_i(t), y_i(t)$  分别表示  $t$  时刻食饵种群和捕食者种群的密度; 系数  $b_i(t), d_i(t), \theta_i(t)$  分别表示食饵的出生率、死亡率和密度制约系数;  $\mu_i(t), \rho_i(t)$  分别表示捕食者的死亡率和密度制约函数;  $e_i(t)x_2(t)/(1+f_1(t)x_2(t)), e_1(t)x_2(t)/(1+f_1(t)x_2(t))$  为饱和项, 意味着互惠作用不可能无休止地增加下去.  $c_i(t)x_i(t)/(\alpha_i(t)+\beta_i(t)x_i(t)+\gamma_i(t)y_i(t))$  分别是捕食者  $y_i(t)$  对食饵种群  $x_i(t)$  的功能反应函数. 系数  $b_i(t), c_i(t), d_i(t), e_i(t), f_i(t), D_i(t), \theta_i(t), \mu_i(t), \rho_i(t)$ ,

$$\begin{cases} x_1'(t) = b_1(t)x_1(t) - d_1(t)x_1(t) - \theta_1(t)x_1^2(t) + \frac{e_1(t)x_2(t)}{1+f_1(t)x_2(t)}x_1(t) - \frac{D_1(t)x_1(t)y_1(t)}{\alpha_1(t)+\beta_1(t)x_1(t)+\gamma_1(t)y_1(t)}, \\ x_2'(t) = b_2(t)x_2(t) - d_2(t)x_2(t) - \theta_2(t)x_2^2(t) + \frac{e_2(t)x_1(t)}{1+f_2(t)x_1(t)}x_2(t) - \frac{D_2(t)x_2(t)y_2(t)}{\alpha_2(t)+\beta_2(t)x_2(t)+\gamma_2(t)y_2(t)}, \\ y_1'(t) = -\mu_1(t)y_1(t) - \rho_1(t)y_1^2(t) + \frac{c_1(t)x_1(t)y_1(t)}{\alpha_1(t)+\beta_1(t)x_1(t)+\gamma_1(t)y_1(t)}, \\ y_2'(t) = -\mu_2(t)y_2(t) - \rho_2(t)y_2^2(t) + \frac{c_2(t)x_2(t)y_2(t)}{\alpha_2(t)+\beta_2(t)x_2(t)+\gamma_2(t)y_2(t)}. \end{cases} \quad (1)$$

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$\alpha_i(t), \beta_i(t), \gamma_i(t)$  均为  $t \in [0, +\infty)$  上连续、有界、严格正的函数, 以上条件  $i=1, 2$ . 为了讨论方便, 我们引入以下记号:

$$(x_1, x_2, y_1, y_2) \in R_+^4 = \{(x_1(t), x_2(t), y_1(t), y_2(t)) \in R^4 : x_i(t) \geq 0, y_i(t) \geq 0, i=1, 2\}.$$

由于生物学意义, 我们只在区域  $\text{Int}R_+^4$  中讨论系统(1), 对模型的解  $(x_1, x_2, y_1, y_2)$  只考虑其具有正初值的情况, 即:

$$x_i(t_0) > 0, y_i(t_0) > 0, i=1, 2, t_0 \geq 0. \quad (2)$$

对于  $R$  上连续有界函数  $f(t)$ , 我们记:

$$f^u := \sup_{t \in R} f(t), f^l := \inf_{t \in R} f(t).$$

## 1.2 定义与引理

**定义 1** 对系统(1), 若存在 1 个紧区域  $\Gamma \subset \text{Int}R_+^4$ , 使得系统的任何满足正初值(2)的解  $(x_1, x_2, y_1, y_2)$ , 最终都进入并保留在紧区域  $\Gamma$  中, 则称系统(1)是一致持久的.

**定义 2** 对系统(1)中的 1 个正解  $(x_1, x_2, y_1, y_2)$ , 既满足 Liapunov 意义下的稳定, 又对于其他正解  $(\bar{x}_1, \bar{x}_2, \bar{y}_1, \bar{y}_2)$  有:

$$\lim_{t \rightarrow +\infty} \left( \sum_{i=1}^2 |x_i(t) - \bar{x}_i(t)| + |y_i(t) - \bar{y}_i(t)| \right) = 0,$$

则称系统(1)的解  $(x_1, x_2, y_1, y_2)$  是全局渐进稳定的.

**引理 1** (Brouwer 不动点定理)假设  $\Omega$  是  $R_+^4$  上的有界闭凸集,  $\tau \in C(\Omega, \Omega)$ , 则存在  $U^* \in \Omega$ , 使得  $\tau U^* = U^*$ .

**引理 2** 设  $f(t)$  是定义在  $[0, +\infty)$  上的非负可积函数, 且在  $[0, +\infty)$  上一致连续, 则  $\lim_{t \rightarrow +\infty} f(t) = 0$ .

## 2 主要结果

### 2.1 持久性

**定理 1**  $R_+^4$  是系统(1)的正向不变集.

**证明** 将系统(1)中各式对  $t$  从  $t_0$  到  $t$  积分, 易

得(3)式.

若有  $x_i(t_0) > 0, y_i(t_0) > 0, i=1, 2$ , 当  $t \geq t_0$  时, 显然有  $x_i(t) > 0, y_i(t) > 0, i=1, 2$ . 故该系统满足正初值的任何解恒正. 由此,  $R_+^4$  是系统(1)的正向不变集.

**命题 1** 若条件:

$$b_i^u - d_i^l + e_i^u / f_i^l > 0, i=1, 2, \quad (4)$$

成立. 则  $\exists T_i > 0, M_i > 0, i=1, 2$ , 当  $t \geq T_i$  时,  $x_i(t) \leq M_i$ , 其中,  $M_i > \bar{M}_i \equiv (b_i^u - d_i^l + e_i^u / f_i^l) / \theta_i^l$ .

**证明** 在系统(1)中, 显然,

$$\begin{aligned} x_i'(t) &\leq b_i(t)x_i(t) - d_i(t)x_i(t) - \theta_i(t)x_i^2(t) + \\ &\quad e_i(t)x_j(t)x_i(t) / (1 + f_i(t)x_j(t)), \\ &\quad i, j=1, 2; i \neq j, \end{aligned}$$

则,

$$\begin{aligned} x_i'(t) &\leq b_i(t)x_i(t) - d_i(t)x_i(t) - \theta_i(t)x_i^2(t) + \\ &\quad e_i(t)x_i(t) / f_i(t) \leq (b_i^u - d_i^l + e_i^u / f_i^l)x_i(t) - \\ &\quad \theta_i^l x_i^2(t). \end{aligned}$$

于是,

$$x_i(t) \leq \frac{b_i^u - d_i^l + e_i^u / f_i^l}{\theta_i^l} := \bar{M}_i.$$

由此可得:

(i) 若  $x_i(t_0) \leq M_i$ , 有  $x_i(t) \leq M_i, (t \geq t_0)$ ;

(ii) 若  $x_i(t_0) > M_i$ , 必存在  $T_i \geq t_0$ , 有  $x_i(T_i) \leq M_i$ . 否则令  $-\delta_i = M_i(b_i^u - d_i^l + e_i^u / f_i^l - \theta_i^l M_i), \delta_i > 0$ , 因此,  $x_i'(t) \leq -\delta_i$ , 两边对  $t$  从  $t_0$  到  $t$  积分, 可得  $x_i(t) \leq x_i(t_0) - \delta_i(t - t_0)$ , 与  $x_i(t) \rightarrow -\infty (t \rightarrow +\infty)$  矛盾. 由(i)可知, 当  $t \geq T_i$  时,  $x_i(t) \leq M_i$ .

**命题 2** 若条件:

$$c_i^u / \beta_i^l - \mu_i^l > 0, i=1, 2, \quad (5)$$

成立, 则  $\exists T_3 > 0, N_i > 0, i=1, 2$ , 当  $t \geq T_3$  时,  $y_i(t) \leq N_i, i=1, 2$ , 其中,  $N_i > \bar{N}_i \equiv (c_i^u / \beta_i^l - \mu_i^l) / \rho_i^l$ .

**证明** 由系统(1)可知:  $y_i'(t) \leq -\mu_i(t)y_i(t) -$

$$\begin{cases} x_1(t) = x_1(t_0) \exp \left\{ \int_{t_0}^t [b_1(s) - d_1(s) - \theta_1(s)x_1(s) + \frac{e_1(s)x_2(s)}{1 + f_1(s)x_2(s)} - \frac{D_1(s)y_1(s)}{\alpha_1(s) + \beta_1(s)x_1(s) + \gamma_1(s)y_1(s)}] ds \right\}, \\ x_2(t) = x_2(t_0) \exp \left\{ \int_{t_0}^t [b_2(s) - d_2(s) - \theta_2(s)x_2(s) + \frac{e_2(s)x_1(s)}{1 + f_2(s)x_1(s)} - \frac{D_2(s)y_2(s)}{\alpha_2(s) + \beta_2(s)x_2(s) + \gamma_2(s)y_2(s)}] ds \right\}, \\ y_1(t) = y_1(t_0) \exp \left\{ \int_{t_0}^t [-\mu_1(s) - \rho_1(s)y_1(s) + \frac{c_1(s)x_1(s)}{\alpha_1(s) + \beta_1(s)x_1(s) + \gamma_1(s)y_1(s)}] ds \right\}, \\ y_2(t) = y_2(t_0) \exp \left\{ \int_{t_0}^t [-\mu_2(s) - \rho_2(s)y_2(s) + \frac{c_2(s)x_2(s)}{\alpha_2(s) + \beta_2(s)x_2(s) + \gamma_2(s)y_2(s)}] ds \right\}. \end{cases} \quad (3)$$

$\rho_i(t)y_i^2(t) + c_i(t)y_i(t) / \beta_i(t) \leq (c_i^u / \beta_i^l - \mu_i^l)y_i(t) - \rho_i^l \cdot y_i^2(t)$ . 于是,  $y_i(t) \leq (c_i^u / \beta_i^l - \mu_i^l) / \rho_i^l := \bar{N}_i$ .

与上述讨论相同: 若  $y_i(t_0) \leq N_i$ , 有  $y_i(t) \leq N_i, t \geq t_0$ ; 若  $y_i(t_0) \geq N_i$ , 必存在  $T_3 > t_0$ , 当  $t \geq T_3$  时, 有  $y_i(t) \leq N_i, (t \geq t_0)$ .

**命题3** 若条件:

$$b_i^l - d_i^u - D_i^u / \gamma_i^l > 0, i = 1, 2, \quad (6)$$

成立, 则  $\exists T_i^* > 0, m_i > 0, i = 1, 2$ , 当  $t \geq T_i^*$  时,  $x_i(t) \geq m_i, i = 1, 2$ , 其中,  $m_i < \bar{m}_i \equiv (b_i^l - d_i^u - D_i^u / \gamma_i^l) / \theta_i^u$ .

**证明** 经简单计算可得:

$$x_i'(t) \geq b_i(t)x_i(t) - d_i(t)x_i(t) - \theta_i(t)x_i^2(t) - D_i(t)x_i(t) / \gamma_i(t).$$

因此,

$$x_i'(t) \geq (b_i^l - d_i^u - D_i^u / \gamma_i^l)x_i(t) - \theta_i^u(t)x_i^2(t), i = 1, 2.$$

于是,  $x_i(t) \geq (b_i^l - d_i^u - D_i^u / \gamma_i^l) / \theta_i^u := \bar{m}_i$ .

(i) 若  $x_i(t_0) \geq m_i$ , 有  $x_i(t) \geq m_i, t \geq t_0$ ;

(ii) 若  $x_i(t_0) < m_i$ , 必存在  $T_i^* \geq t_0$ , 有  $x_i(T_i^*) \geq m_i$ . 否则, 令  $\varphi_i = b_i^l - d_i^u - D_i^u / \gamma_i^l - \theta_i^u m_i > 0, x_i'(t) \geq \varphi_i x_i(t)$ , 两边对  $t$  从  $t_0$  到  $t$  积分, 可得  $x_i(t) \geq x_i(t_0) \exp\{\varphi_i(t - t_0)\}$ , 与  $x_i(t) \rightarrow +\infty (t \rightarrow +\infty)$  矛盾. 由(i)知, 当  $t \geq T_i^*$  时,  $x_i(t) \geq m_i$ .

**命题4** 若条件:

$$z_i := c_i^l m_i / (\alpha_i^u + \beta_i^u m_i + \gamma_i^u N_i) - \mu_i^u > 0, \quad i = 1, 2, \quad (7)$$

成立, 则  $\exists T_4 > 0, n_i > 0, i = 1, 2$ , 当  $t \geq T_4$  时,  $y_i(t) \geq n_i, i = 1, 2$ , 其中,  $n_i < \bar{n}_i \equiv z_i / \rho_i^u$ .

**证明** 易知:

$$y_i'(t) \geq \frac{c_i(t)m_i y_i(t)}{\alpha_i(t) + \beta_i(t)m_i + \gamma_i(t)N_i} - \mu_i(t)y_i(t) - \rho_i(t)y_i^2(t) \geq (\frac{c_i^l m_i}{\alpha_i^u + \beta_i^u m_i + \gamma_i^u N_i} - \mu_i^u) \cdot y_i(t) - \rho_i^u(t)y_i^2(t) = z_i y_i(t) - \rho_i^u(t)y_i^2(t).$$

因此,  $y_i(t) \geq z_i / \rho_i^u := \bar{n}_i$ . 与前文相同, 若  $y_i(t_0) \geq n_i$ , 有  $y_i(t) \geq n_i, t \geq t_0$ ; 当  $y_i(t_0) < n_i$  时, 必存在  $T_4 \geq t_0$ , 当  $t \geq T_4$  时, 有  $y_i(t) \geq n_i, t \geq t_0$ .

**定理2** 若系统(1)满足条件(6)式和(7)式, 则该系统一致持久.

**证明** 由条件(6)式和(7)式不难推出条件(4)式和(5)式. 根据前面的证明, 可知:

$$\exists m, M > 0,$$

使得:

$$0 < m = \min\{m_i, n_i\} \leq \inf_{t \rightarrow +\infty} \{x_i(t), y_i(t)\} \leq$$

$$\sup_{t \rightarrow +\infty} \{x(t), y(t)\} \leq \max\{M_i, N_i\} = M, i = 1, 2,$$

其中,  $m_i, n_i, M_i, N_i$  均为前文所述. 故  $\Omega = [m, M] \times [m, M] \times [m, M] \times [m, M]$  为该系统的正向不变集以及最终有界域, 所以该系统一致持久.

## 2.2 全局渐进稳定性

**定理3** 对于系统(1),  $(x_1^*, x_2^*, y_1^*, y_2^*)$  是它的1个有界正解, 若满足条件(6)式和(7)式, 且满足:

$$\left\{ \begin{array}{l} \inf_{t \in R} [d_i - \frac{D_i \beta_i N_i + c_i \alpha_i + c_i \gamma_i N_i}{(\alpha_i + \beta_i m_i + \gamma_i n_i)^2} - \frac{e_j}{(1 + f_j m_i)^2}] > 0, \\ \inf_{t \in R} [\mu_i - \frac{D_i \alpha_i + D_i \beta_i M_i}{(\alpha_i + \beta_i m_i + \gamma_i n_i)^2} + \frac{c_i \gamma_i m_i}{(\alpha_i + \beta_i M_i + \gamma_i N_i)^2}] > 0, \end{array} \right. \quad i, j = 1, 2; i \neq j. \quad (8)$$

则系统(1)的解  $(x_1^*, x_2^*, y_1^*, y_2^*)$  是全局渐进稳定的. 这里  $m_i, n_i, M_i, N_i$  均为前文所述. 为了方便性, 此处,  $b_i, c_i, d_i, e_i, f_i, D_i, \theta_i, \mu_i, \rho_i, \alpha_i, \beta_i, \gamma_i$  分别代表  $b_i(t), c_i(t), d_i(t), e_i(t), f_i(t), D_i(t), \theta_i(t), \mu_i(t), \rho_i(t), \alpha_i(t), \beta_i(t), \gamma_i(t)$ .

**证明** 令  $(x_1, x_2, y_1, y_2)$  为该系统的任一具正初值的解, 则  $\exists T > 0$ , 当  $t \geq t_0 + T$  时,  $(x_1, x_2, y_1, y_2) \in \Omega$ , 定义:

$$V(t) = \sum_{i=1}^2 (|\ln x_i - \ln x_i^*| + |\ln y_i - \ln y_i^*|),$$

则,

$$\begin{aligned} D^+ V(t) = & \operatorname{sgn}\{x_1 - x_1^*\} \left[ -\left( \frac{e_1 x_2^*}{1 + f_1 x_2^*} - \frac{e_1 x_2}{1 + f_1 x_2} \right) - \right. \\ & d_1(x_1 - x_1^*) - \left( \frac{D_1 y_1}{\alpha_1 + \beta_1 x_1 + \gamma_1 y_1} - \frac{D_1 y_1^*}{\alpha_1 + \beta_1 x_1^* + \gamma_1 y_1^*} \right) \left. \right] + \operatorname{sgn}\{x_2 - x_2^*\} \left[ -\left( \frac{e_2 x_1^*}{1 + f_2 x_1^*} - \frac{e_2 x_1}{1 + f_2 x_1} \right) - \right. \\ & d_2(x_2 - x_2^*) - \left( \frac{D_2 y_2}{\alpha_2 + \beta_2 x_2 + \gamma_2 y_2} - \frac{D_2 y_2^*}{\alpha_2 + \beta_2 x_2^* + \gamma_2 y_2^*} \right) \left. \right] + \operatorname{sgn}\{y_1 - y_1^*\} \left[ -\mu_1(y_1 - y_1^*) - \left( \frac{c_1 x_1^*}{\alpha_1 + \beta_1 x_1^* + \gamma_1 y_1^*} - \frac{c_1 x_1}{\alpha_1 + \beta_1 x_1 + \gamma_1 y_1} \right) \right] + \\ & \operatorname{sgn}\{y_2 - y_2^*\} \left[ -\mu_2(y_2 - y_2^*) - \right. \end{aligned}$$

$$\begin{aligned}
& \left( \frac{c_2 x_2^*}{\alpha_2 + \beta_2 x_2^* + \gamma_2 y_2^*} - \frac{c_2 x_2}{\alpha_2 + \beta_2 x_2 + \gamma_2 y_2} \right) \leq \\
& \frac{e_1 |x_2^* - x_2|}{(1 + f_1 m_2)^2} - d_1 |x_1 - x_1^*| - \operatorname{sgn}\{x_1 - x_1^*\} \cdot \\
& \frac{D_1 y_1 \alpha_1 + D_1 y_1 \beta_1 x_1^* - D_1 y_1^* \alpha_1 - D_1 y_1^* \beta_1 x_1}{(\alpha_1 + \beta_1 x_1 + \gamma_1 y_1)(\alpha_1 + \beta_1 x_1^* + \gamma_1 y_1^*)} + \\
& \frac{e_2 |x_1^* - x_1|}{(1 + f_2 m_1)^2} - d_2 |x_2 - x_2^*| - \operatorname{sgn}\{x_2 - x_2^*\} \cdot \\
& \frac{D_2 y_2 \alpha_2 + D_2 y_2 \beta_2 x_2^* - D_2 y_2^* \alpha_2 - D_2 y_2^* \beta_2 x_2}{(\alpha_2 + \beta_2 x_2 + \gamma_2 y_2)(\alpha_2 + \beta_2 x_2^* + \gamma_2 y_2^*)} - \\
& \mu_1 |y_1 - y_1^*| - \operatorname{sgn}\{y_1 - y_1^*\} \cdot \\
& \frac{c_1 x_1^* \alpha_1 + c_1 x_1^* \gamma_1 y_1 - c_1 x_1 \alpha_1 - c_1 x_1 \gamma_1 y_1^*}{(\alpha_1 + \beta_1 x_1^* + \gamma_1 y_1^*)(\alpha_1 + \beta_1 x_1 + \gamma_1 y_1)} - \\
& \mu_2 |y_2 - y_2^*| - \operatorname{sgn}\{y_2 - y_2^*\} \cdot \\
& \frac{c_2 x_2^* \alpha_2 + c_2 x_2^* \gamma_2 y_2 - c_2 x_2 \alpha_2 - c_2 x_2 \gamma_2 y_2^*}{(\alpha_2 + \beta_2 x_2^* + \gamma_2 y_2^*)(\alpha_2 + \beta_2 x_2 + \gamma_2 y_2)} \leq \\
& \frac{e_1 |x_2^* - x_2|}{(1 + f_1 m_2)^2} - d_1 |x_1 - x_1^*| + (D_1 \alpha_1 |y_1 - y_1^*| + \\
& \operatorname{sgn}\{x_1 - x_1^*\} D_1 \beta_1 (y_1^* x_1 - y_1^* x_1^* + y_1^* x_1^* - \\
& y_1 x_1^*)) / ((\alpha_1 + \beta_1 x_1 + \gamma_1 y_1)(\alpha_1 + \beta_1 x_1^* + \\
& \gamma_1 y_1^*)) + \frac{e_2 |x_1^* - x_1|}{(1 + f_2 m_1)^2} - d_2 |x_2 - x_2^*| + (D_2 \alpha_2 \cdot \\
& |y_2 - y_2^*| + \operatorname{sgn}\{x_2 - x_2^*\} D_2 \beta_2 (y_2^* x_2 - y_2^* x_2^* + \\
& y_2^* x_2^* - y_2 x_2^*)) / ((\alpha_2 + \beta_2 x_2 + \gamma_2 y_2)(\alpha_2 + \\
& \beta_2 x_2^* + \gamma_2 y_2^*)) - \mu_1 |y_1 - y_1^*| + (c_1 \alpha_1 |x_1 - x_1^*| + \\
& \operatorname{sgn}\{y_1 - y_1^*\} c_1 \gamma_1 (x_1 y_1^* - x_1 y_1 + x_1 y_1 - x_1^* y_1^*)) / \\
& ((\alpha_1 + \beta_1 x_1^* + \gamma_1 y_1^*)(\alpha_1 + \beta_1 x_1 + \gamma_1 y_1)) - \\
& \mu_2 |y_2 - y_2^*| + (c_2 \alpha_2 |x_2 - x_2^*| + \operatorname{sgn}\{y_2 - \\
& y_2^*\} c_2 \gamma_2 (x_2 y_2^* - x_2 y_2 + x_2 y_2 - x_2^* y_2^*)) / \\
& ((\alpha_2 + \beta_2 x_2^* + \gamma_2 y_2^*)(\alpha_2 + \beta_2 x_2 + \gamma_2 y_2)) \leq \\
& \frac{e_1 |x_2^* - x_2|}{(1 + f_1 m_2)^2} - d_1 |x_1 - x_1^*| + (D_1 \alpha_1 |y_1 - y_1^*| + \\
& D_1 \beta_1 N_1 |x_1 - x_1^*| + D_1 \beta_1 M_1 |y_1 - y_1^*|) / (\alpha_1 + \\
& \beta_1 m_1 + \gamma_1 n_1)^2 + \frac{e_2 |x_1^* - x_1|}{(1 + f_2 m_1)^2} - d_2 |x_2 - x_2^*| + \\
& (D_2 \alpha_2 |y_2 - y_2^*| + D_2 \beta_2 N_2 |x_2 - x_2^*| + \\
& D_2 \beta_2 M_2 |y_2 - y_2^*|) / (\alpha_2 + \beta_2 m_2 + \gamma_2 n_2)^2 - \\
& \mu_1 |y_1 - y_1^*| + \frac{c_1 \alpha_1 |x_1 - x_1^*| + c_1 \gamma_1 N_1 |x_1 - x_1^*|}{(\alpha_1 + \beta_1 m_1 + \gamma_1 n_1)^2} -
\end{aligned}$$

$$\begin{aligned}
& \frac{c_1 \gamma_1 m_1 |y_1 - y_1^*|}{(\alpha_1 + \beta_1 M_1 + \gamma_1 N_1)^2} - \mu_2 |y_2 - y_2^*| + \\
& \frac{c_2 \alpha_2 |x_2 - x_2^*| + c_2 \gamma_2 N_2 |x_2 - x_2^*|}{(\alpha_2 + \beta_2 m_2 + \gamma_2 n_2)^2} - \\
& \frac{c_2 \gamma_2 m_2 |y_2 - y_2^*|}{(\alpha_2 + \beta_2 M_2 + \gamma_2 N_2)^2} \leq \\
& -[d_1 - \frac{D_1 \beta_1 N_1 + c_1 \alpha_1 + c_1 \gamma_1 N_1}{(\alpha_1 + \beta_1 m_1 + \gamma_1 n_1)^2} - \\
& \frac{e_2}{(1 + f_2 m_1)^2}] |x_1 - x_1^*| - [d_2 - \\
& \frac{D_2 \beta_2 N_2 + c_2 \alpha_2 + c_2 \gamma_2 N_2}{(\alpha_2 + \beta_2 m_2 + \gamma_2 n_2)^2} - \frac{e_1}{(1 + f_1 m_2)^2}] \cdot \\
& |x_2 - x_2^*| - [\mu_1 - \frac{D_1 \alpha_1 + D_1 \beta_1 M_1}{(\alpha_1 + \beta_1 m_1 + \gamma_1 n_1)^2} + \\
& \frac{c_1 \gamma_1 m_1}{(\alpha_1 + \beta_1 M_1 + \gamma_1 N_1)^2}] |y_1 - y_1^*| - \\
& [\mu_2 - \frac{D_2 \alpha_2 + D_2 \beta_2 M_2}{(\alpha_2 + \beta_2 m_2 + \gamma_2 n_2)^2} + \\
& \frac{c_2 \gamma_2 m_2}{(\alpha_2 + \beta_2 M_2 + \gamma_2 N_2)^2}] |y_2 - y_2^*|.
\end{aligned}$$

由条件可知, 存在 1 个  $\nu > 0$ , 使得:

$$\begin{aligned}
D^+V(t) & \leq -\nu \left\{ \sum_{i=1}^2 (|x_i - x_i^*| + |y_i - y_i^*|) \right\}, \\
t & \geq t_0 + T.
\end{aligned}$$

两边对  $t$  从  $t_0 + T$  到  $t$  积分, 可得:

$$\begin{aligned}
V(t) + \nu \int_{t_0+T}^t \left\{ \sum_{i=1}^2 (|x_i - x_i^*| + |y_i - y_i^*|) \right\} ds & \leq \\
V(t_0 + T) & < +\infty, t \geq t_0 + T.
\end{aligned}$$

因此,

$$\begin{aligned}
\limsup_{t \rightarrow +\infty} \int_{t_0+T}^t \left\{ \sum_{i=1}^2 (|x_i - x_i^*| + |y_i - y_i^*|) \right\} ds & \leq \\
\frac{V(t_0 + T)}{\nu} & < +\infty.
\end{aligned}$$

所以  $\sum_{i=1}^2 (|x_i - x_i^*| + |y_i - y_i^*|) \in L^1([t_0 + T, +\infty))$ .

因此,  $\sum_{i=1}^2 (|x_i - x_i^*| + |y_i - y_i^*|)$  在  $[t_0 + T, +\infty)$  上一致连续且非负可积.

由引理 2 知,  $\lim_{t \rightarrow +\infty} \sum_{i=1}^2 (|x_i - x_i^*| + |y_i - y_i^*|) = 0$ . 因此, 从定义 2 直接能得到  $(x_1^*, x_2^*, y_1^*, y_2^*)$  是全局渐进稳定的.

### 2.3 正周期解的存在唯一性和全局渐进稳定性

在实际的生态系统中, 不能忽略季节更替、食物供给及种群繁殖习性对物种造成的影响. 而这

些影响往往是呈周期性变化的,假设系统(1)中的所有系数都是以 $\omega$ 为周期的连续正的函数,则该系统成为了一个 $\omega$ -周期系统.

**定理 4** 若系统(1)满足条件(6)式、(7)式及(8)式,则该系统在 $R_+^4$ 中存在唯一的正的周期解,且全局渐进稳定.

**证明** 定义 1 个 Poincare 映射:

$$P(U_0) = U(t_0, U_0): R_+^4 \rightarrow R_+^4,$$

其中,  $U_0 = (x_1(t_0), x_2(t_0), y_1(t_0), y_2(t_0))$ ,  $U(t, t_0, U_0)$  是系统(1)过正初值点 $(t_0, U_0)$ 的解,取定理 2 中的有界闭凸集 $\Omega$ ,算子 $P$ 映射 $\Omega$ 到自身,由该系统的解对初值的连续依赖性知算子 $P$ 也是连续的,因此由引理 1 可知,在 $\Omega$ 中至少存在 1 个不动点 $U^*$ ,  $U^* = (x_1^*(t), x_2^*(t), y_1^*(t), y_2^*(t)) \in \Omega$ . 所以,系统(1)存在 1 个正周期解.由定理 2 和定理 3 可知其周期解是唯一的,而且全局渐进稳定.

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## Permanence and Stability of a Two-predator-and-two-cooperative-prey System

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**Abstract:** In this article, a two-predator-and-two-cooperative-prey model with Beddington-DeAngelis functional response for the predator is investigated. Using comparison theorem, the permanence of the system is obtained. Through constructing a Lyapunov function, the global asymptotic stability of the system is proved under some appropriate restraints. Further, for the periodic case, a set of sufficient conditions is presented, which guarantees the existence, uniqueness and global asymptotic stability of a positive periodic solution.

**Key words:** mutualistic; predator-prey system; functional response; permanence; stability

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