Existence of Periodic Solution for a Predator-prey System with Feedback Control

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Abstract: A Hassell-Varely type predator-prey system with feedback controls is studied. Using the comparison, continuation theorems and coincidence degree theorem, the existence of positive periodic solutions for the system is proven. Also, a set of sufficient conditions for global stability is derived through constructing a Lyapunov function.

Key words: Hassell-Varely type; predator-prey; coincidence degree theorem; Lyapunov function; global stability

CLC number: O175 **Document code:** A **Article ID:** 1001-5132（2012）02-0051-06

In 1969, Hassell and varley [1] introduced a general predator-prey system, in which the functional response dependents on the predator density in different way. It is called a Hassell-varley (HV for short) type functional response which take the following form:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = rx(1-\dfrac{x}{K}) - \dfrac{cxy}{my^{\gamma}+x}, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = y(-d+\dfrac{fx}{my^{\gamma}+x}), \end{cases} \quad \gamma \in (0,1), \tag{1}$$

where γ is called the HV constant. In the typical predator-prey interaction where predators do not form groups, one can assume that terrestrial $\gamma = 1$, producing the so-called ratio-dependent PP system. For terrestrial predators that form a fixed number of tight groups, it is often reasonable to assume $\gamma = 1/2$. For aquatic predators that form a fixed number of tights groups, $\gamma = 1/3$ may be more appropriate. A unified mechanistic approach was provided by Cosner [2], where the HV functional response was derived. Hsu [3] studied system (1) and presented a systematic global qualitative analysis to it.

However, the logistic growth does not fit well for some populations. For example, the Gompertz growth $x' = x\ln(K/x)$ provides an excellent fit to empirical growth curves for avascular tumors and vascular tumors in their early stages [4-5]. Indeed, many data have shown that per capita growth functions of populations are well fitted by logarithmic regressions, for example, the Gompertz model has been almost universally used to descrybe the growth of microorganisms [6-7], some creature [8-9], and the innovation diffusion such as digital cellular telephones [10-11], and the references cited therein.

Motivated by the above reasons, we consider a HV type predator-prey model with controls,

$$\begin{cases} x_1'(t) = x_1(t)[a_1(t) - b_1(t)\ln x_1(t) - \\ \qquad \dfrac{c_1(t)x_2(t)}{mx_2^{\gamma}(t)+x_1(t)} - d_1(t)u_1(t)], \\ x_2'(t) = x_2(t)[a_2(t) - b_2(t)\ln x_2(t) + \\ \qquad \dfrac{c_2(t)x_1(t)}{mx_2^{\gamma}(t)+x_1(t)} - d_2(t)u_2(t)], \\ u_1'(t) = \alpha_1(t) - \beta_1(t)u_1(t) + \gamma_1(t)x_1(t), \\ u_2'(t) = \alpha_2(t) - \beta_2(t)u_2(t) + \gamma_2(t)x_2(t), \end{cases} \quad \gamma \in (0,1), \tag{2}$$

Received date: 2011−06−28. JOURNAL OF NINGBO UNIVERSITY (NSEE): http://3xb.nbu.edu.cn
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where $x_i(t), i=1,2$ denote the density of prey and predator at time t, $u_i(t), i=1,2$ are control variables, $a_i, b_i, c_i, d_i, \alpha_i, \beta_i, \gamma_i \in (R, R_+), i=1,2$ are all w-periodic functions of t and m is a nonnegative constant.

For convenience, throughout this paper, we shall use the following notations. For a continuous w-periodic functions $f(t)$,

$$f^M = \max_{t\in[0,w]} f(t),\ f^L = \min_{t\in[0,w]} f(t),\ \overline{f} = \frac{1}{w}\int_0^w f(t)\mathrm{d}t .$$

Throughout this paper, we suppose that the following conditions are satisfied:

(F1) $a_i, b_i, c_i, d_i, \alpha_i, \beta_i, \gamma_i \in (R, R_+), i=1,2$ are nonnegative w-periodic functions of t.

(F2) $a_i, \beta_i \in (R, R), i=1,2$ are w-periodic functions with $a_i^L > 0, \beta_i^L > 0$.

The organization of this paper is as follows. In the next section, we discuss the existence of positive w-periodic solutions of system (2). In section 3, by constructing a Lyapunov functional, we establish a sufficient condition for the global stability of w-periodic solutions of system (2).

1 Existence of positive periodic solutions

Let X and Y be normed vector spaces. Let $L: \mathrm{Dom}L \subset X \to Y$ be a linear mapping and $N: X \to Y$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim\mathrm{Ker}L = \mathrm{codim}\,\mathrm{Im}\,L < \infty$ and $\mathrm{Im}\,L$ is closed in Y. If L is a Fredholm mapping of index zero, then there exist continuous projectors $P: X \to X$ and $Q: Y \to Y$ such that $\mathrm{Im}P = \mathrm{Ker}L$ and $\mathrm{Im}\,L = \mathrm{Ker}Q = \mathrm{Im}(I-Q)$. It follows that $L \mid \mathrm{Dom}L \cap \mathrm{Ker}P: (I-P)X \to \mathrm{Im}\,L$ is invertible and its inverse is denoted by K_P. If Ω is a bounded open subset of X, the mapping N is called $L-$ compact on $\overline{\Omega}$, if $QN(\overline{\Omega})$ is bounded and $KP(I-Q)N: \overline{\Omega} \to X$ is compact. Because $\mathrm{Im}\,Q$ is isomorphic to $\mathrm{Ker}L$, there exists an isomorphism $J: \mathrm{Im}\,Q \to \mathrm{Ker}L$. In the proof of our existence result, we need the following continuation theorem.

Lemma 1[12] Let $L: \mathrm{Dom}L \subset X \to Y$ be the Fredholm injection with index of 0, $N: \overline{\Omega} \to Y$ on $\overline{\Omega}L$-compact. Suppose $\lambda \in (0,1)$ holds, then $Lx \neq \lambda Nx$; Suppose $x \in \mathrm{Ker}L \cap \partial\Omega$, $QNx \neq 0$ holds, and $\deg(JQN|_{\mathrm{Ker}L\cap\partial\Omega}) \neq 0$, where $\deg(JQN|_{\mathrm{Ker}L\cap\partial\Omega})$, represents the Brouwer degree. Then $Lx = Nx$ has at least one solution on $\mathrm{Dom}L \cap \overline{\Omega}$.

Lemma 2[13] Suppose $\phi(t)$ is continuous differenttial w-periodic function. Then

$$|\phi(t_1) - \phi(t_2)| \leqslant |\sup_{s\in[0,w]} \phi(s) - \inf_{s\in[0,w]} \phi(s)| \leqslant$$

$$\frac{1}{2}\int_0^w |\phi'(s)|\,\mathrm{d}s, t_1, t_2 \in [0.w].$$

Theorem 1 Under the assumptions of (F1), (F2). Then system (2) has at least one positive w-periodic solution.

Proof Let $x_i(t) = \exp(y_i(t)), i=1,2, u_1(t) = y_3(t)$, $u_2(t) = y_4(t)$, then the system (2) becomes

$$\begin{cases} y_1'(t) = a_1(t) - b_1(t)y_1(t) - \dfrac{c_1(t)e^{y_2(t)}}{me^{\gamma y_2(t)} + e^{y_1(t)}} - \\ \qquad d_1(t)y_3(t) \equiv f_1, \\ y_2'(t) = a_2(t) - b_2(t)y_2(t) + \dfrac{c_2(t)e^{y_1(t)}}{me^{\gamma y_2(t)} + e^{y_1(t)}} - \\ \qquad d_2(t)y_4(t) \equiv f_2, \\ y_3'(t) = \alpha_1(t) - \beta_1(t)y_3(t) + \gamma_1(t)e^{y_1(t)} \equiv f_3, \\ y_4'(t) = \alpha_2(t) - \beta_2(t)y_4(t) + \gamma_2(t)e^{y_2(t)} \equiv f_4. \end{cases} \quad (3)$$

In order to use Lemma 1 to system (3), we set $X = \{Y(t) = (y_1(t), y_2(t), y_3(t), y_4(t))^{\mathrm{T}}: y_i(t) \in PC_W, i=1,2,3,4\} = Y$, with norm $\|Y\| = \|(y_1(t), y_2(t), y_3(t), y_4(t))^{\mathrm{T}}\| = \sum_{i=1}^{i=4}\|y_i(t)\| = \sum_{i=1}^{i=4}\max_{0\leqslant t\leqslant w}|y_i(t)|$, then $(X, \|.\|)$ is a Banach space.

Set $L: DomL \subset X \to Y$, as $L\begin{pmatrix} y_1 \\ \vdots \\ y_4 \end{pmatrix} = \begin{pmatrix} y_1'(t) \\ \vdots \\ y_4'(t) \end{pmatrix}$,

where $DomL = \{Y(t) = (y_1, y_2, y_3, y_4)^{\mathrm{T}} \in X \mid Y'(t) \in PC_W\} = \{Y(t) = (y_1, y_2, y_3, y_4)^{\mathrm{T}} \in X \mid Y(t) \in PC_W^{'}\}$.

At the same time, we denote $N(y(t)) = \begin{pmatrix} f_1 \\ \vdots \\ f_4 \end{pmatrix}$, and define two projectors P and Q as $P: X \to X$,

$$P(Y(t)) = \frac{1}{w}\begin{pmatrix} \int_0^w y_1(t)\mathrm{d}t \\ \vdots \\ \int_0^w y_4(t)\mathrm{d}t \end{pmatrix};$$

$Q: Y \to Y$, as

$$Q\begin{pmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \\ f_4(t) \end{pmatrix} = \frac{1}{w}\begin{pmatrix} \int_0^w y_1(t)\mathrm{d}t \\ \vdots \\ \int_0^w y_4(t)\mathrm{d}t \end{pmatrix}.$$

Obviously,

$$\operatorname{Im} L = \left\{ z = \begin{pmatrix} f_1(t) \\ \vdots \\ f_4(t) \end{pmatrix} \in Y : \int_0^w f_i(t)\mathrm{d}t = 0, i = 1,2,3,4 \right\},$$

and

$$\operatorname{Ker} L = \left\{ x : x \in X, y \in \begin{pmatrix} e_1 \\ \vdots \\ e_4 \end{pmatrix} \in R^4 \right\} = \operatorname{Im} P,$$

$\operatorname{Im} L = \{x \in Y : \int_0^w f_i(t)\mathrm{d}t = 0, i = 1,2,3,4\} = \operatorname{Ker} Q$, are closed sets in Y and $\dim \operatorname{Ker} L = \operatorname{codim} \operatorname{Im} L = 4$.

Hence, L is a Fredholm mapping of index zero. Furthermore, the generalized inverse of $L: K_P: \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$ has the form

$$K_P(z) = K_P\begin{pmatrix} y_1'(t) \\ y_2'(t) \\ y_3'(t) \\ y_4'(t) \end{pmatrix} = \begin{pmatrix} \int_0^t f_1(s)\mathrm{d}s - \frac{1}{w}\int_0^w \int_0^t f_1(s)\mathrm{d}s\mathrm{d}t \\ \vdots \\ \int_0^t f_4(s)\mathrm{d}s - \frac{1}{w}\int_0^w \int_0^t f_4(s)\mathrm{d}s\mathrm{d}t \end{pmatrix}.$$

Thus, $QN(Y(t)) = \frac{1}{w}\begin{pmatrix} \int_0^w f_1(t)\mathrm{d}t \\ \vdots \\ \int_0^w f_4(t)\mathrm{d}t \end{pmatrix}$, and

$$K_P(I-Q)N(y(t)) = \begin{pmatrix} \int_0^t f_1(t)\mathrm{d}t \\ \vdots \\ \int_0^t f_4(t)\mathrm{d}t \end{pmatrix} + \left(\frac{1}{2} - \frac{t}{w}\right)\begin{pmatrix} \int_0^w f_1(t)\mathrm{d}t \\ \vdots \\ \int_0^w f_4(t)\mathrm{d}t \end{pmatrix} - \frac{1}{w}\begin{pmatrix} \int_0^w \int_0^t f_1(s)\mathrm{d}s\mathrm{d}t \\ \vdots \\ \int_0^w \int_0^t f_4(s)\mathrm{d}s\mathrm{d}t \end{pmatrix}.$$

By Lebesque convergence theorem, we know QN and $K_P(I-Q)N(\overline{\Omega})$ are continuous. Moreover, by Lemma 1, we get $QN(\overline{\Omega}), K_P(I-Q)N(\overline{\Omega})$ are relatively compact for any open bounded set $\Omega \subset X$. Hence, N is L-compact on $\overline{\Omega}$, here Ω is any open bounded set in X.

Now we are in a position to search for an appropriate open bounded subset Ω for the application of Lemma 1, corresponding to equation $Lx = \lambda Nx$, $\lambda \in (0,1)$, we have

$$y_i'(t) = \lambda f_i(t), i = 1,2,3,4. \tag{4}$$

Suppose that $Y(t) = (y_1(t), y_2(t), y_3(t), y_4(t))^{\mathrm{T}} \in X$ is a periodic solution of system (3) for a certain $\lambda \in (0,1)$, By integrating system (4) over the interval [0,w], we can obtain

$$\int_0^w y_i'(t)\mathrm{d}t = \lambda \int_0^w f_i(t)\mathrm{d}t,\ i = 1,2,3,4. \tag{5}$$

Thus,

$$\begin{cases} \int_0^w \{a_1(t) - b_1(t)y_1(t) - \frac{c_1(t)e^{y_2(t)}}{me^{\gamma y_2(t)} + e^{y_1(t)}} - \\ \quad d_1(t)y_3(t)\}\mathrm{d}t = 0, \\ \int_0^w \{a_2(t) - b_2(t)y_2(t) + \frac{c_2(t)e^{y_1(t)}}{me^{\gamma y_2(t)} + e^{y_1(t)}} - \\ \quad d_2(t)y_4(t)\}\mathrm{d}t = 0, \\ \int_0^w \{\alpha_1(t) - \beta_1(t)y_3(t) + \gamma_1(t)e^{y_1(t)}\}\mathrm{d}t = 0, \\ \int_0^w \{\alpha_2(t) - \beta_2(t)y_4(t) + \gamma_2(t)e^{y_2(t)}\}\mathrm{d}t = 0. \end{cases} \tag{6}$$

Note that $Y(t) = (y_1(t), y_2(t), y_3(t), y_4(t))^{\mathrm{T}} \in X$, then there exist $\xi_i, \eta_i \in [0,w]$, $i = 1,2,3,4$, such that

$$y_i(\xi_i) = \inf_{t\in[0,w]} y_i(t), \quad y_i(\eta_i) = \sup_{t\in[0,w]} y_i(t), \quad i = 1,2,3,4. \tag{7}$$

It follows from Eq. (5) and Eq. (6) that $\int_0^w b_1(t)y_1(t)\mathrm{d}t \leqslant \int_0^w a_1(t)\mathrm{d}t$, thus $y_1(\xi_1) \leqslant \frac{\overline{a_1}}{\overline{b_1}}$.

By Lemma 2, the comparison theorem and the first Eq. (6), we have

$$y_1(\eta_1) \leqslant y_1(\xi_1) + \frac{1}{2}\int_0^w |y_1'(t)|\mathrm{d}t \leqslant \frac{\overline{a_1}}{\overline{b_1}} + \overline{a_1}w \equiv A_1. \tag{8}$$

Similarly, we have

$$y_2(\xi_2) \leqslant \frac{\overline{a_2} + \overline{c_2}}{\overline{b_2}},$$

$$y_2(\eta_2) \leqslant y_2(\xi_2) + \frac{1}{2}\int_0^w |y_2'(t)|\mathrm{d}t \leqslant \frac{\overline{a_2} + \overline{c_2}}{\overline{b_2}} + (\overline{a_2} + \overline{c_2})w \equiv A_2,$$

$$y_3(\xi_3) \leqslant \frac{\overline{\alpha_1} + \overline{\gamma_1}e^{A_1}}{\overline{\beta_1}},$$

$$y_3(\eta_3) \leqslant y_3(\xi_3) + \frac{1}{2}\int_0^w |y_3'(t)|\mathrm{d}t \leqslant$$

$$\frac{\overline{\alpha_1}+\overline{\gamma_1}e^{A_1}}{\overline{\beta_1}}+(\overline{\alpha_1}+\overline{\gamma_1}e^{A_1})w\equiv A_3,$$

$$y_4(\xi_4)\leqslant\frac{\overline{\alpha_2}+\overline{\gamma_2}e^{A_2}}{\overline{\beta_2}},$$

$$y_4(\eta_4)\leqslant y_4(\xi_4)+\frac{1}{2}\int_0^w|y_4'(t)|\mathrm{d}t\leqslant$$

$$\frac{\overline{\alpha_2}+\overline{\gamma_2}e^{A_2}}{\overline{\beta_2}}+(\overline{\alpha_2}+\overline{\gamma_2}e^{A_2})w\equiv A_4.$$

On the other hand, by Eq. (6) and comparison theorem, we get

$$\overline{b_1}y_1(\eta_1)w\geqslant\int_0^w b_1(t)y_1(t)\mathrm{d}t\geqslant\int_0^w\{a_1(t)-$$

$$\frac{c_1(t)e^{y_2(t)-\gamma y_2(t)}}{m}-d_1(t)y_3(\eta_3)\}\mathrm{d}t\geqslant$$

$$\int_0^w\{a_1(t)-\frac{c_1(t)e^{(1-\gamma)y_2(\eta_2)}}{m}-d_1(t)y_3(\eta_3)\}\mathrm{d}t\geqslant$$

$$y_1(\eta_1)\geqslant\frac{\overline{a_1}-\frac{\overline{c_1}e^{(1-\gamma)A_2}}{m}-\overline{d_1}A_3}{\overline{b_1}}.$$

Thus by Lemma 2, we obtain

$$y_1(\xi_1)\geqslant y_1(\eta_1)-\frac{1}{2}\int_0^w|y_1'(t)|\mathrm{d}t\geqslant$$

$$\frac{\overline{a_1}-\frac{\overline{c_1}e^{(1-\gamma)A_2}}{m}-\overline{d_1}A_3}{\overline{b_1}}-\overline{a_1}w\equiv B_1.$$

Similarly, we have

$$y_2(\xi_2)\geqslant y_2(\eta_2)-\frac{1}{2}\int_0^w|y_2'(t)|\mathrm{d}t\geqslant$$

$$\frac{\overline{a_2}-\overline{d_2}A_4}{\overline{b_2}}-(\overline{a_1}+\overline{c_2})w\equiv B_2,$$

$$y_3(\xi_3)\geqslant y_3(\eta_3)-\frac{1}{2}\int_0^w|y_3'(t)|\mathrm{d}t\geqslant$$

$$\frac{\overline{\alpha_1}+\overline{\gamma_1}e^{B_1}}{\overline{\beta_1}}-(\overline{\alpha_1}+\overline{\gamma_1}e^{A_1})w\equiv B_3,$$

$$y_4(\xi_4)\geqslant y_4(\eta_4)-\frac{1}{2}\int_0^w|y_4'(t)|\mathrm{d}t\geqslant$$

$$\frac{\overline{\alpha_2}+\overline{\gamma_2}e^{B_2}}{\overline{\beta_2}}-(\overline{\alpha_2}+\overline{\gamma_2}e^{A_2})w\equiv B_4.$$

We let $M_i=\max\{|B_i|,|A_i|\}$, $i=1,2,3,4$, respectively. It is obvious that $M_i, i=1,2,3,4$ are independent of λ. Similar to the proof of Theorem 3.1 of [14], we can find a sufficiently large $M>0$ denote the set $\Omega=\{Y(t)=(y_1(t),y_2(t),y_3(t),y_4(t))\in X:\|Y\|\leqslant M\}$.

It follows that for each $u\in\mathrm{Ker}L\cap\partial\Omega, QNu\neq 0$ and $\deg\{JQNu,\Omega\cap\mathrm{Ker}L,0\}\neq 0$.

By now we have proved that Ω verifies all the requirements in Lemma 1. Hence Eq. (2) has at least one positive w-periodic solution. The proof is complete.

2 Uniqueness and global stability

We proceed to the discussion on the uniqueness and global stability of the w-periodic solution $x^*(t)$ in Theorem 1. It is immediate that if $x^*(t)$ is golbally asymptotically stable then $x^*(t)$ is unique in fact.

Lemma 3 [15] Let f be a nonnegative function defined on $[0,+\infty)$ such that f is integrable on $[0,+\infty)$ and is uniformly continuous on $[0,+\infty)$. Then $\lim_{t\to+\infty}f(t)=0$.

Theorem 2 let $X(t)=(x_1(t),x_2(t),u_1(t),u_2(t))^{\mathrm{T}}$ denote any positive solution of Eq. (2) , then there exists a $T>0$ such that if $t>T$, $m_i<x_i(t)<M_i, i=1,2,$ $m_3<u_1(t)<M_3, m_4<u_2(t)<M_4$.

Proof It follows from the positivity of the solution of Eq. (2) that $x_1'(t)\leqslant x_1(t)[a_1(t)-b_1(t)\ln x_1(t)]$.

A standard comparison argument shows that

$$\limsup_{t\to\infty}x_1(t)\leqslant\exp(\frac{a_1^M}{b_1^L})\equiv M_1.$$

Hence, there exists a $T_1>0$ such that if $t>T_1$, $x_1(t)\leqslant M_1$. According to the second equation of system (2), we have

$$x_2'(t)\leqslant x_2(t)[a_2(t)-b_2(t)\ln x_2(t)+\frac{c_2(t)x_1(t)}{mx_2^\gamma(t)+x_1(t)}].$$

Thus there exist a $T_2>T_1$ such that

$$x_2(t)\leqslant\exp(\frac{a_2^M+c_2^M M_1}{b_2^L})\equiv M_2, t>T_2.$$

For the third Eq. (2), we have

$$u_1'(t)\leqslant\alpha_1^M+\gamma_1^M M_1-\beta_1^L u_1(t).$$

Thus, there exist a $T_3>T_2$ such that

$$u_1(t)\leqslant\frac{\alpha_1^M+\gamma_1^M M_1}{\beta_1^L}\equiv M_3, t>T_3.$$

Similarly, there exist a $T_4>T_3$ such that

$$u_2(t)\leqslant\frac{\alpha_2^M+\gamma_2^M M_2}{\beta_2^L}\equiv M_4, t>T_4.$$

On the other hand, according to the second equation of system (2), we have $x_2'(t)\geqslant x_2(t)[a_2(t)-$

$b_2(t)\ln x_2(t)-d_2(t)u_2(t)]\geqslant x_2(t)[a_2^L-b_2^M\ln x_2(t)-d_2^M M_4]$.

Thus there exist a $T_5>T_4$ such that

$$x_2(t)\geqslant \exp(\frac{a_2{}^L-d_2{}^M M_4}{b_2{}^M})\equiv m_2, t>T_5,$$

for the first equation of system (2) , we have

$$x_1'(t)\geqslant x_1(t)[a_1^L-b_1^M\ln x_1(t)-\frac{c_1{}^M M_2}{mm_2{}^\gamma}-d_1{}^M M_3].$$

Then there exist $T_6>T_5$ such that

$$x_1(t)\geqslant \exp(\frac{a_1{}^L-\frac{c_1{}^M M_2}{mm_2{}^\gamma}-d_1{}^M M_3}{b_1{}^M})\equiv m_1, t>T_6.$$

For the third equation of system (2) we have

$$u_1'(t)\geqslant \alpha_1{}^L+\gamma_1{}^L m_1-\beta_1{}^M u_1(t).$$

Thus, there exist $T_7>T_6$ such that

$$u_1(t)\geqslant \frac{\alpha_1{}^L+\gamma_1{}^L m_1}{\beta_1{}^M}\equiv m_3, t>T_7.$$

Similarly, there exist $T_8>T_7$ such that

$$u_2(t)\geqslant \frac{\alpha_2{}^L+\gamma_2{}^L m_2}{\beta_2{}^M}\equiv m_4, t>T_8.$$

Thus, the proof is complete.

We now formulate the global stability of the positive w-periodic solution of system (2).

Theorem 3 In addition to (F1) and (F2), assume further that $\liminf_{t\to+\infty} A_i(t)>0$, $i=1,2,3,4$, where

$$A_1(t)=\frac{b_1(t)}{M_1}-\frac{c_1(t)(M_2-2m_2)}{m^2m_2{}^{2\gamma}}-\frac{mc_2(t)M_2{}^\gamma}{m^2m_2{}^\gamma}-\gamma_1(t),$$

$$A_2(t)=\frac{b_2(t)}{M_2}-\frac{c_2(t)(2mM_2{}^\gamma+M_1-m(\gamma+1)m_2{}^\gamma)}{m^2m_2{}^{2\gamma}}+\frac{c_2(t)m\gamma m_1m_2{}^{\gamma-1}}{(M_1+mM_2{}^\gamma)^2}-\gamma_2(t), \tag{9}$$

$$A_3(t)=\beta_1(t)-d_1(t),$$

$$A_4(t)=\beta_2(t)-d_2(t).$$

Then system (2) has a unique positive w-periodic solution which is globally asymptotically stable.

Proof Let $x^*(t)=(x_1^*(t),x_2^*(t),u_1^*(t),u_2^*(t))^{\mathrm T}$ be a positive w-periodic solution of system (2), and $y(t)=(y_1(t),y_2(t),w_1(t),w_2(t))^{\mathrm T}$ be any positive solution of system (2). It follows from Theorem 2 that exist positive constant T and M_i, m_i, such that for all $t>T$, $m_i<x_i^*(t)<M_i, i=1,2, m_3<u_1^*(t)<M_3, m_4<u_2^*(t)<M_4$, $m_i<y_i(t)<M_i, i=1,2, m_3<w_1(t)<M_3, m_4<w_2(t)<M_4$.

We define $V_1(t)=|\ln x_1^*(t)-\ln y_1(t)|$, calculating the upper right derivative of $V_1(t)$ along solutions of Eq. (2):

$$\begin{aligned}D^+V_1(t)&=(\frac{x_1^{*\prime}(t)}{x_1^*(t)}-\frac{y_1'(t)}{y_1(t)})\mathrm{sgn}(x_1^*(t)-y_1(t))=\\&\mathrm{sgn}(x_1^*(t)-y_1(t))\{-b_1(t)\ln x_1^*(t)+b_1(t)\cdot\\&\ln y_1(t)-\frac{c_1(t)x_2^*(t)}{mx_2^{*\gamma}(t)+x_1^*(t)}+\frac{c_1(t)y_2(t)}{my_2{}^\gamma(t)+y_1(t)}-\\&d_1(t)u_1^*(t)+d_1(t)w_1(t)\}\leqslant-\frac{b_1(t)}{M_1}|x_1^*(t)-\\&y_1(t)|+\frac{c_1(t)(2mM_2{}^\gamma+M_1-m(\gamma+1)m_2{}^\gamma)}{m^2m_2{}^\gamma}\cdot\\&|x_2^*(t)-y_2(t)|+\frac{c_1(t)(M_2-2m_2)}{m^2m_2{}^\gamma}|x_1^*(t)-\\&y_1(t)|+d_1(t)|u_1^*(t)-w_1(t)|.\end{aligned}$$

Similarly, we define $V_2(t)=|\ln x_2^*(t)-\ln y_2(t)|$, thus,

$$\begin{aligned}D^+V_2(t)&=(\frac{x_2^{*\prime}(t)}{x_2^*(t)}-\frac{y_2'(t)}{y_2(t)})\mathrm{sgn}(x_2^*(t)-y_2(t))\leqslant\\&-\frac{b_2(t)}{M_2}|x_2^*(t)-y_2(t)|-\frac{c_2(t)m\gamma m_1m_2{}^{\gamma-1})}{(M_1+mM_2{}^\gamma)^2}\cdot\\&|x_2^*(t)-y_2(t)|+\frac{c_2(t)mM_2{}^\gamma}{m^2m_2{}^\gamma}|x_1^*(t)-y_1(t)|+\\&d_2(t)|u_2^*(t)-w_2(t)|.\end{aligned}$$

Then we define:

$$V_3(t)=|u_1^*(t)-w_1(t)|,$$

$$V_4(t)=|u_2^*(t)-w_2(t)|,$$

$$D^+V_3(t)\leqslant-\beta_1(t)|u_1^*(t)-w_1(t)|+\gamma_1(t)|x_1^*(t)-y_1(t)|,$$

$$D^+V_4(t)\leqslant-\beta_2(t)|u_2^*(t)-w_2(t)|+\gamma_2(t)|x_2^*(t)-y_2(t)|.$$

We now define a Lyapunov functional $V(t)$ as

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t),$$

then we can get for $t>T$,

$$\begin{aligned}D^+V(t)\leqslant&-A_1(t)|x_1^*(t)-y_1(t)|-A_2(t)|x_2^*(t)-\\&y_2(t)|-A_3(t)|u_1^*(t)-w_1(t)|-\\&A_4(t)|u_2^*(t)-w_2(t)|.\end{aligned}$$

Where $A_i(t)>0, i=1,2$ are defined in Theorem 3.

By hypothesis, there exist positive constant $\alpha_i, i=1,2,3,4$ and $T^*\geqslant T$ such that if $t\geqslant T^*$.

$$A_i(t)\geqslant\alpha_i>0. \tag{10}$$

Integrating both sides of Eq. (10) on interval $[T^*,t]$,

$$V(t)+\sum_{i=1}^{2}\int_{T^*}^{t}A_i(s)\,|\,x_i^*(s)-y_i(s)\,|\,\mathrm{d}s+\int_{T^*}^{t}A_3(s)\,|\,u_1^*(s)-w_1(s)\,|\,\mathrm{d}s+\int_{T^*}^{t}A_4(s)\,|\,u_2^*(s)-w_2(s)\,|\,\mathrm{d}s\leqslant V(T^*),$$

thus,

$$V(t)+\sum_{i=1}^{2}\int_{T^*}^{t}\alpha_i\,|\,x_i^*(s)-y_i(s)\,|\,\mathrm{d}s+\int_{T^*}^{t}\alpha_3\,|\,u_1^*(s)-w_1(s)\,|\,\mathrm{d}s+\int_{T^*}^{t}\alpha_4\,|\,u_2^*(s)-w_2(s)\,|\,\mathrm{d}s\leqslant V(T^*)$$

$t>T^*$.

Then by Lemma 3, we have

$$\lim_{t\to\infty}|\,x_i^*(t)-y_i(t)\,|=0,\lim_{t\to\infty}|\,u_i^*(t)-w_i(t)\,|=0, i=1,2.$$

Which implies the positive w-periodic solution of system (2) is globally asymptotically stable. The proof is complete.

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一类带有回馈控制的捕食-食饵模型正周期解的存在性

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摘要： 研究了一类具有 Hassell-Varely 功能性反应函数的食饵-捕食模型的回馈控制系统，利用比较连续定理和一致度定理，证明了系统正周期解的存在性，并通过构造 Lyapunov 函数给出了系统全局稳定性的充分条件和证明.

关键词： Hassell-Varely 功能反应函数；食饵-捕食系统；一致度定理；全局稳定性

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