Hierarchic Nature of Dynamic Deformation and Fracture of Rock Mass

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Abstract: Dynamic deformation and fracture process of rock mass are investigated from the perspective of structural hierarchy. The relationship between spatial scales of deformation and fracture with strain rate is explored in the framework of relaxation model. The velocity of crack propagation in dependence on loading intensity is discussed, and the hierarchic nature of deformation and fracture are examined. At last the fracture criteria are discussed from the aspects concerning structural hierarchy of rock mass. The investigation shows that dynamic deformation and fracture process of rock mass possess hierarchic nature which depends on the spatial and temporal characteristics of external loading, structural hierarchy of rock mass and finiteness of the velocity of the fracture process. The temporal criterion and limit strain criterion may serve as fracture criteria adequately well.

Key words: hierarchy of internal structure; rock mass; dynamic deformation and fracture; hierarchic nature

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1 Introduction

Generally in continuum mechanics solid is represented as ideal continuum medium whose deformation under external forces is completely reversible if the internal stresses in solid don’t exceed their limit values characterizing the strength of solid.

But real materials have complex internal structure which has decisive effect on mechanical behavior of materials. For rock mass an important peculiarity of such discreteness is the similarity of the internal structure in a wide range of sizes. Investigation showed that [1], a fundamental canonical series for the sizes Δ_i of geo-blocks exists:

$$\Delta_i = (\sqrt{2})^{-i}\Delta_0, \tag{1}$$

where $\Delta_0 = 2.5\times 10^6$ m is the radius of Earth’s core; i is positive integers.

According to the investigation in [2], the ratio of openings of cracks δ_i to linear size of blocks Δ_i separated by these cracks at i-th scale level is stable, and can be described by the next relation that has a normal statistical distribution

$$\mu_\Delta(\delta) = \frac{\delta_i}{\Delta_i} = \Theta 10^{-2}, \tag{2}$$

where Θ is a coefficient changing in the interval 1/2 – 2, and parameter μ_Δ is termed as “geo-mechanical

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invariant” in [2].

The structural hierarchy of rocks specifies the hierarchy of deformation and fracture processes. Slow processes, such as energy storage between earthquakes at the geotectonic levels as well as the diffusion stress waves, tectonic solitary waves with different periods, rotary waves, seismicity caused by the oil and gas production, and the construction of petroleum storage reservoirs etc., commonly occur in the hierarchical levels with geo-blocks of great sizes[3-4].

The fast processes, for instance, the deformation and fracture of materials under shock loading[5], are related to the mesoscopic or microscopic structural levels. The interim processes, including micro-earthquakes, rock-bursts, and so on, occur at levels between the tectonic and meso-microscopic levels[6].

It has been from the conceptions of the structural hierarchy that scientists have been studying the laws of deformation and fracture of geo-medium at different structural levels since the 1970’s. For example, the relations between the sizes of geo-blocks and the fracture times were examined in [3,7]. The comparisons between the stiffness of inter-block layers and the thickness of geo-block layers were analyzed in [8-9]. The relations between the structural hierarchy, the viscosity, and the strength of geo-blocks were investigated in [10-11]. However, the hierarchic nature of deformation and fracture of rock mass are still not fully understood, the essence of the relations between spatial and temporal scales is not revealed completely, the deformation and fracture processes of rock mass from the viewpoint of structural hierarchy is not fully clarified. The present paper is devoted to the investigation of the above mentioned problems in order to reveal the hierarchic nature of deformation and fracture of rock mass.

2 The relationship between temporal and spatial scales of deformation and fracture of rock mass

The internal structure has decisive impact on mechanical behavior of rock mass. If the strength of crystals with ideal regular lattices is their theoretical strength, then the strength of real materials is about 2 – 3 orders lower than their theoretical strength. Obviously, the complex hierarchic internal structure of real materials will causes the stress concentration and strain localization which are responsible for lowering of real material strength. Accordingly the stresses in heterogeneous solids may be looked as being composed of two components[6]: elastic stresses induced by the reversible volume and shear deformations, and the local inelastic stresses in heterogeneities which are responsible for the irreversible deformations. The elastic stresses are related to the reversible deformations linearly. As to the residual stresses (inelastic stresses), they arise at definite strain rate, and relax with time. The evolution equation for the residual stress deviator Δs_{ij}^{l} in heterogeneities with characteristic linear size l may be described by Maxwell model

$$\frac{\mathrm{d}\Delta s_{ij}^{l}}{\mathrm{d}t} = 2\rho c_s^2 \dot{e}_{ij} - v\frac{\Delta s_{ij}^{l}}{l}, \tag{3}$$

where Δs_{ij}^{l} is the residual stress deviator components in heterogeneities with characteristic scale l; $\dot{e}_{ij}$ is the residual strain rate deviator components; ρ is the density of the medium; v is the relaxation velocity; c_s is the elastic shear wave propagation velocity. Here we suppose that all residual stress deviator components relax with the same relaxation time.

The main feature of this model is that, the relaxation rate of the residual deviator stresses in heterogeneities is proportional to the magnitude of the residual stresses, and inversely proportional to the size of the heterogeneities.

The solution of equation (3) has the following form:

$$\Delta s_{ij}^{l} = 2\rho c_s^2 \dot{e}_{ij}\frac{l}{v}(1 - e^{-vt/l}), \tag{4}$$

with the time the residual deviator stresses approach their stationary values

$$\Delta s_{ij}^{l} = 2\rho c_s^2 \dot{e}_{ij}\frac{l}{v}. \tag{5}$$

Substituting equation (5) into the expression $\Delta\sigma_I = \sqrt{3\Delta s_{ij}^l \Delta s_{ij}^l / 2}$, for intensity of residual stress deviator, we obtain

$$\Delta\sigma_I = 3\rho c_s^2 \dot{e}_{ij} \frac{l}{v}, \tag{6}$$

where $\dot{\varepsilon} = \sqrt{2\dot{e}_{ij}\dot{e}_{ij}/3}$, is the residual strain rate intensity.

It can be seen from equations (5) and (6) that at given strain rate the greater the size of the heterogeneities is, the greater the residual stresses are. If the size of the body is infinite, then we can always find large enough heterogeneities that their residual stresses are large enough to cause the fracture of the body. If we denote the limit residual stress causing fracture of the body as σ^*, then we can determine the minimal size of heterogeneities corresponding to the limit residual stress σ^*:

$$l = \frac{\sigma^*}{3\rho c_s^2} \frac{v}{\dot{\varepsilon}}. \tag{7}$$

In this way, at constant strain rate among the parameters of solid a parameter with dimension of length arises.

It can be seen from equation (7) that, for any body we can find one strain rate that does not make the body fractured. In the frame of this model such a behavior of solid corresponds to creep.

3 Hierarchic natures of deformation and fracture of rock mass

Now let us analyze equation (7). For plastically non-compressible rock mass Poisson’s ratio $\mu = 0$, and Young’s elastic modulus is $E = 2(1+\mu)G = 2(1+0.5)\cdot \rho c_s^2 = 3\rho c_s^2$. Therefore in one-dimensional case the term $\sigma^*/(3\rho c_s^2)$ on the right of equation (7) represents the magnitude ε^* of deformation corresponding to strength limit σ^*, and the term $\sigma^*/(3\rho c_s^2\dot{\varepsilon})$ denotes the time for strain to reach deformation ε^* at constant strain rate $\dot{\varepsilon}$: $\sigma^*/(3\rho c_s^2\dot{\varepsilon}) = \varepsilon^*/\dot{\varepsilon} = \tau^*$. v may be looked at as the propagation velocity of crack. Therefore equation (7) may be rewritten as

$$l = \frac{\sigma^*}{3\rho c_s^2} \frac{v}{\dot{\varepsilon}} = \frac{\varepsilon^*}{\dot{\varepsilon}} v = \tau^* v, \tag{8}$$

which denotes the propagation distance of crack at the moment of failure.

Propagation velocity v of crack depends on loading conditions. Experimental observations of fracture propagation[12-13] indicate that crack may grow for energy lower than the critical limit of fracture. At micro-scale, the tensile failure due to the sub-critical propagation of cracks may represent the main micro-mechanism of creep at the macro-scale. The dependence of fracture propagation rate on the stress intensity factor in mode I may be approximated by tri-modal behavior, which is shown in Fig. 1.

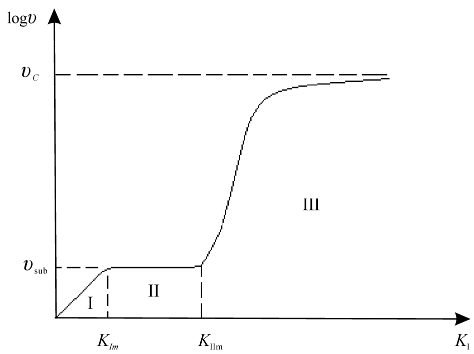

Fig. 1　The dependence of crack propagation velocity on stress intensity factor

In region I, the rate of stress corrosion reaction controls the velocity of crack growth. Region II is mainly determined by the rate of transport of reactive species to crack tips. In region III, the velocity of crack growth increases drastically up to failure and is relatively independent of the chemical environment and is controlled by mechanical rupture.

Considering crack growth as a succession of water vapor enhanced rupture of small material element immediately adjacent to the crack tips, Salganik et al[14] showed that the three regimes of crack propagation can be predicted, respectively, by three following equations:

$$v_I = \frac{v_0 \varphi_0^n}{n_{mol}} \exp\left(-\frac{U'_{01} - \Omega_1' K_1 / \sqrt{2\pi d_m}}{kT}\right), \tag{9}$$

$$v_{II} = \frac{b_0 p}{n_{mol} N_A \sqrt{2\pi mkT}}, \tag{10}$$

$$v_{III} = \frac{d_m}{\tau_0} \exp(-\frac{U'_{03} - \Omega'_3 K_1 / \sqrt{2\pi d_m}}{kT}), \quad (11)$$

where T is the absolute temperature; τ_0 is the typical period of atomic fluctuations; v_0 is the mean velocity of diffusion; φ_0 is the relative concentration of water in the gas next to the crack tips; n_{mol} is the number of molecules of water required for the water assisted rupture of a single bridging bonding; d_m is the length of the material structure; b_0 is the bridging bond length; N_A is the Avogadro number; m is the molecular mass of water; p is the partial pressure of water vapor; U'_{01} and U'_{03} are the zero stress activation energy in regions I and III, respectively; Q'_1 and Q'_3 are stress sensitivity factors in regions I and III, respectively.

Experiments show that the limit crack growth velocity is about 0.4 of Reyleigh wave speed C_R [15]. Recently the relationship between limit crack growth velocity v_c and the mechanical characteristics of material has been obtained theoretically in [16]

$$v_c(R)=((1-\gamma)/(4(1+\gamma)((\gamma^2-3\lambda/2+7/8)\cdot \ln(2R/a)-3\gamma/8+3/32+a^2/(8R^2))))^{1/2}\cdot (E/\rho)^{1/2}, \quad (12)$$

where R is the size of region of the material involved in the crack propagation; γ is the Poisson's ratio; E is Young's elastic modulus; ρ is the material density; a is the half-length of the crack. For example, for $R=2.6a$, $\gamma=0.25$, equation (21) gives $v_c=0.38\sqrt{E/\rho}$.

Therefore there are two characteristic crack propagation velocities: the crack growth velocity corresponding to plateau II v_{sub} and limit crack growth velocity v_C.

According to equation (2), at every structural level the thickness of the weakened surfaces is proportional to the characteristic size of the elements at given structural level. It is natural to assume that the smaller the thickness of the structural surfaces is, i.e. the smaller the character size of this structural level is, the higher the mechanical characteristics of rock mass at this level are.

The magnitude of strain rates is related to the stress wave profile. The rise phase of stress wave is the main phase that induces dynamic response of materials. The shorter the rise time t_r is, and the greater the stress magnitude is, the higher is the strain rate. If we denote the wave propagation velocity in material as c, the more the strain rate is, the less is the region ct_r covered by the rise phase of the wave, and the higher is the strength of material.

From other hand, in the region ct_r covered by the rise phase of stress wave, because of the finiteness of the wave propagation velocity $v \leqslant v_C$, for 0-th level elements with maximum size $D_0 \sim ct_r$, the strength of structural surface of material is σ_{N0}, the entire time for structural surface to open is $t_0 = \sigma_{N0} / (\dot{\varepsilon}\rho c^2) + D_0 / v$. If the growth of loading is so faster that before the complete failure of 0-th level elements the magnitude of stress reaches the strength of the 1-th level element with size $D_1 < D_0$, in this way the inter-element structural surfaces of 1-th level elements start to crack. The time necessary for 1-th level inter- element surfaces to failure completely is $t_1 = \sigma_{N1} / (\dot{\varepsilon}\rho c^2) + D_1 / v$.

If before the complete failure of 0-th and 1-th level elements loading continues to grow, then the initiation of cracking of inter-element surfaces of the next smaller scale level elements is activated and so on. Thus the strength of inter-element structural surfaces of the least scale elements at moment of macroscopic failure is the dynamic strength of material.

In this way we can clearly see the intrinsic relationship between size and strain rate effects. The fracture time is the minimum of the quantity $\sigma_{Ni} / (\dot{\varepsilon}\rho c^2) + D_i / v$ for all involved structural levels, i.e.

$$\tau_{frac} = \min(\frac{\sigma_{Ni}}{\dot{\varepsilon}\rho c^2} + D_i / v). \quad (13)$$

It is clear that the term $\sigma_{Ni} / (\dot{\varepsilon}\rho c^2)$ in the right-hand side of equation (13) is related to the elastic behavior of material, and the second term D_i / v represents the relaxation time at i-th scale level.

4 On fracture criteria of rock mass

In static loading conditions or at low strain rates cracks at the largest scale level have time to develop, and the strength of the structural surfaces at this level controls the static strength σ_{c0} of rock mass. Therefore "force" criteria are good enough for the description of the fracture. At high strain rates the situation becomes much complicated. If the strain rates and the maximum attained stresses are high enough, then before the fracture of rock mass at the largest scale level deformation and fracture processes at smaller scale levels will be activated. Thus "force" criteria are not enough for describing the deformation and fracture, they are only necessary conditions for fracture of rock mass. In order that fracture takes place at *i*-th scale level the loading time must exceeds the time D_i / v necessary for the cracks to propagate through out the elements at this level. From this viewpoint the concept of incubation time for fracture proposed by Morozov N F and Petrov Y V[17] is appropriate. Obviously, incubation time t_{inc}^{i} for fracture at *i*-th scale level is $t_{inc}^{i} = D_i / v$, the product $\sigma_i^i t_{inc}^i$ of the strength σ_{ci} and incubation time t_{inc} at *i*-th scale level constitutes the critical value of impulse for the fracture of rock mass.

Nikiforovsky-Shemyakin's impulse criterion[18] supposes that, fracture occurs when the integral of local stress with time, i.e. when the local stress $\sigma(t)$ impulse exceeds one critical value J_c, i.e.

$$\int_0^{t_*} \sigma(t)\mathrm{d}t \geqslant J_c. \tag{14}$$

According the concept of incubation time proposed by Morozov N F and Petrov Y V, equation (14) should be rewritten as

$$\int_0^{t_*} \sigma(t)\mathrm{d}t \geqslant \sigma_{c0} t_{inc}. \tag{15}$$

In solids the next relation holds

$$\sigma = \rho D v, \tag{16}$$

where ρ is the density of materials; v is the velocity of particle of materials; and D is the velocity of shock wave. Hence for pulse criterion (14), we have

$$\int_0^{t_*} \sigma(t)\mathrm{d}t = \int_0^{t_*} \rho D v \mathrm{d}t = \rho D u = J_c, \tag{17}$$

where u is particle displacement.

Formula (17) shows that, when macroscopic displacement of particles reaches critical value, fracture occurs.

Displacement is the macroscopic measure of deformation of materials. If the characteristic dimension covered by shock wave is L_{shock}, then displacement can be expressed by deformation ε as

$$u = L_{shock}\varepsilon. \tag{18}$$

Therefore formula (17) becomes

$$\int_0^{t_*} \sigma(t)\mathrm{d}t = \rho D L_{shock}\varepsilon = J_c, \tag{19}$$

i.e. when deformation of solids reaches critical value, fracture occurs.

But with the increase of strain rate more scale levels are involved into deformation and fracture processes, the critical strain magnitude should increase. For example if shock wave covers the *j*-th scale level element, and if only the structural surfaces at this level are fractured, then the limit strain at *j*-th scale level should be close to "geo-mechanical invariant"

$$\varepsilon_{jcr} \approx \mu_\Delta. \tag{20}$$

Experimental data conform this conclusion[6].

If the next smaller scale level, i.e. (*j*-1)-th level, is also activated, and $\Delta_{j-1} / \Delta_j = \alpha < 1$, then the limit strain will be

$$\varepsilon_{jcr} \approx \frac{\mu_\Delta \Delta_j + \alpha \mu_\Delta \Delta_j}{\Delta_j} = (1+\alpha)\mu_\Delta. \tag{21}$$

If the next two smaller scale levels: (*j*-1)-th and (*j*-2)-th levels are activated, then the limit strain will be

$$\varepsilon_{jcr} \approx (1+\alpha+\alpha^2)\mu_\Delta, \tag{22}$$

and so on.

Experiments show that the limit failure strain really increases with the strain rate[19]. The rock-like material, concrete, also shows the same strain rate sensitivity of limit failure strain[20].

With more and more scale levels being activated, the limit strain will approach to the following limit

$$\varepsilon_{jcr} \to \frac{\mu_\Delta}{1-\alpha}. \tag{23}$$

For example, if $\alpha = 1/\sqrt{2}$, then $\varepsilon_{jcr} \approx 3.4\mu_\Delta$, which agrees with the experimental data in [19-20].

The relative stable nature of limit failure strain allows us to use limit strain criterion as an alternative fracture criterion for rock mass together with temporal fracture criteria. The essence of temporal criteria and limit strain criterion is that the material must has enough time to develop deformation before limit failure strain is attained.

5 Conclusions

Rock mass has complex internal structural hierarchy, at every scale level mechanical properties of materials are different. Such structural hierarchy specifies the hierarchic nature of deformation and fracture of rock mass. According to the temporal and spatial properties of loading different scale levels are activated in the processes of deformation and fracture of rock mass. In the present paper the relationship between spatial and temporal scales of deformation and fracture is studied from the viewpoint of structural hierarchy. It is shown that the relationship between spatial scales and temporal scales of deformation and fracture of rock mass is determined by the structural hierarchy of rock mass and the limitness of crack propagation. The essence of strain rate effect of strength is that because of the limitness of crack propagation velocity the increase of loading activates the deformation and fracture processes at smaller scale levels before the macro-fracture of the body, the dynamic strength of materials is the structural surface strength of the smallest activated structural elements before the macro- fracture of the sample. It is also shown that temporal criteria and limit strain criterion may serve as favorable fracture criteria for rock mass, The essence of temporal criteria and limit strain criterion is that the material must has enough time to develop deformation before limit failure strain is attained.

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岩体动力变形破坏的层次特性

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摘要：岩体具有复杂的内部结构，内部结构对于岩土的力学性质具有决定性的影响. 文章从岩体结构层次的角度研究了岩体的动力变形与破坏过程，在松弛模型的框架内研究了变形破坏的空间尺度与应变率之间的关系. 讨论了裂纹传播速度与荷载强度之间的关系，研究了岩体变形破坏的层次特性. 最后从岩体结构层次角度研究了岩体的破坏准则. 研究表明：岩体的动力变形与破坏具有层次特性，这一层次特性依赖于外载的空间与时间特性、岩体的结构层次和岩体变形与破坏过程速度的有限性. 时间准则与极限变形准则可以较好地描述岩体的动力破坏.

关键词：内部结构层次；岩体；动力变形与破坏；层次特性

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