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Incubation-time Based Approach for Dynamic Yielding of Metals

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Abstract: Transition of the materials from elastic to plastic state is modeled by incubation-time based criterion. This approach is similar to the structural-temporal theory for brittle solids fracture developed in St.-Petersburg State University. This approach allow us to model anomalous behavior of yield stress in case of elevated temperature, brittle-to-ductile failure mode transition and some other effects.

Key words: dynamic yielding; incubation-time approach; failure criteria

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The structural-temporal theory of quasi-brittle fracture of solids had been proposed in 1980s. Later it was developed to the incubation-time approach that proved to be an effective tool of analysis for various phenomena: ductile or brittle failure of defect-free specimen, fracture of specimen with macro-defect (crack), plastic yielding, cavitation in liquids, electrical breakdown in solids, phase transition etc. Some principal results obtained during two last decades are presented in recent monograph [1]. New results concerning dynamic yielding of metals are to be discussed below.

1 Dynamic yield criterion for metals and alloys

Dependence of yield stress on loading time had been observed in numerous experiments during last decades. This dependence is usually treated as a "strain-rate effect". Although experimental investigations show that this approach cannot be applied for explanation of some phenomena such as "yield delay". This phenomenon is described in paper [2] and in many other works. The applied stress is supported on the constant level. Despite the fact that the values of applied stress is significantly exceed static yield limit the yielding occurs not immediately but after a definite time ("yield delay"). This delay tends to grow with decreasing of temperature (Fig. 1) and for low temperature it may reach the values of several seconds. Dependence of yield delay (t_*) on applied stress (σ_*) is in a good agreement with following scaling law:

$$\log t_* - \alpha \log \sigma_* = Const . \tag{1}$$

Eq. (1) corresponds to straight lines in logarithmic scales; the slopes of these lines are defined by different values of the shape parameter α that significantly decreases with growth of temperature (Fig. 1). The Eq. (1) proved to be valid for numerous experimental data

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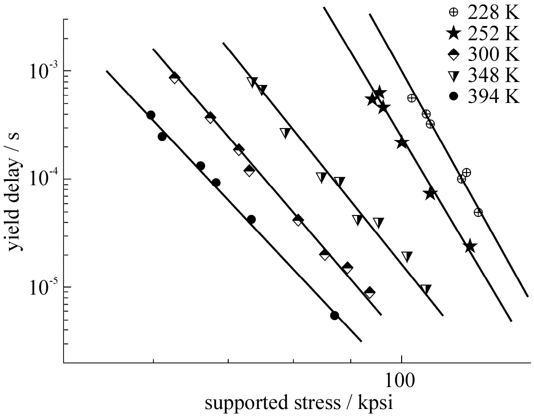


Fig. 1 Dependence of yield delay on applied stress for mild steel (C 0.22, Mn 0.36). Experimental data from [2]

obtained using different loading history.

The necessity to take into consideration a real loading history was emphasized by many famous researchers [3]. Various applications to engineering problems, analysis and comparison of experimental data obtained by different techniques require yield criterion applicable for arbitrary loading history. Such criterion was proposed in our previous works [4-5]:

$$\frac{1}{\tau_Y} \cdot \int\limits_{t-\tau_Y}^{t} \left(\frac{\sigma(s)}{\sigma_Y} \right)^{\alpha} \mathrm{d}s < 1 , \tag{2}$$

where τ_Y incubation time of yielding, σ_Y static yield limit and α shape parameter. They are considered as the material parameters. The transition to the plastic state corresponds to earliest violation of the condition (2). It is easy to show that for loading pulse of short duration the criterion (2) is in a good correspondence with Eq. (1) while in case of slow loading the condition (2) is in close agreement with quasistatic yield criterion

$$\sigma(t) < \sigma_Y . \tag{3}$$

Therefore criterion (2) is valid in a very wide range of loading rates [4-7].

2 Time-temperature correspondence

The duration of loading (loading rate) is not the only parameter that influence the processes discussed. One of the most important parameters is the temperature during testing. The material characteristics (τ_Y, σ_Y, α) proved to be strongly dependent on temperature. Analyzing experimental data for various materials we propose approximations for these dependencies.

$$\sigma_Y = \sigma_0 \cdot \exp\left(-\frac{T}{T_P} \right), \tag{4}$$

$$\tau_Y = \tau_0 \exp\left(\frac{U}{kT} \right), \tag{5}$$

$$\alpha = \alpha_0 \left(1 - \exp\left(-\frac{W}{kT} \right) \right), \tag{6}$$

where T_P, σ_0, τ_0, U, α_0, W are material constants. Further investigation have shown [6-7] that dependencies Eqs (4-6) are enough good for many materials (different kinds of steel, molybdenum, niobium, titanium alloys, aluminum etc.) in wide range of temperatures and strain rate.

It necessary to note that at present time thermal effect on yielding is often considered in very simplified manner. Many existing models consider only thermal softening, i.e. decreasing of yield with increasing of temperature (Eq. (4)). The other approach is based on the concept of “rheologically simple material” [3]. For such materials the change of temperature is equivalent to change of time scale (Eq. (5)). The example of such approach is well-known hypothesis of equivalency between increasing of strain-rate and decreasing of temperature.

Our results demonstrate that such simplified approaches are unsatisfactory for the most of investigated materials. The Eq. (6) demonstrates that the most of metals and alloys couldn’t be considered as “rheologically simple material”. The change of temperature not only shift diagram $\sigma_* - \dot{\varepsilon}$ vertically (thermal softening) and horizontally (time-temperature correspondence), but also radically changes the shape of these diagrams.

The change of shape of the diagram $\sigma_* - \dot{\varepsilon}$ can lead to reverse temperature dependence of yield stress in the range of very high strain-rate: the dynamic yield stress for elevated temperatures may be higher than for low temperatures. This phenomenon contradicts to the concept of thermal softening but it is predicted by our model. The experimental confirmation of such “anomalous” behavior of yield stress was obtained for pure titanium and single-crystal aluminum [7].

One of the possible applications of yield criterion is the prediction of the conditions of brittle-to-ductile transition in fracture of solids. Brittle-to-ductile transition is usually considered as the result of competition between two mechanisms of deformation and fracture. The assumption that these processes can be analyzed separately is known as Ludwik-Davidenko-Orowan hypothesis. Using simultaneously criterion of plastic yielding (2) and similar criterion of brittle fracture [1], we can estimate the time necessary for each of the processes (“fracture delay” and “yield delay”). Hence we are able to predict which of them (cleavage or plastic yielding) should take place earlier.

Temperature and strain-rate are usually considered as the most important parameters influencing the failure mode. It was as early as 1930s when Davidenko N N and his collaborators (Vitmann V F, Stepanov V I, et al.) had shown that increasing of strain-rate shifts the transition temperature to the domain of higher temperatures. But recently obtained results on the behavior of materials in case of extremely high strain-rate (anomalous behavior of yield limit etc.) require renewing some traditional views.

Eqs. (2,4-6) allow us to calculate dependence of yield stress on both-strain-rate and temperature, i.e. $\sigma_*(\dot{\varepsilon},T)$. Similar dependence can be obtained for resistance to brittle fracture. Intersection of these surfaces gives us a curve in plane $T-\dot{\varepsilon}$ that separate domains of brittle and ductile fracture. This curve can be treated as a strain-rate dependence of transitional temperature. Preliminary calculations for some metals show that this dependence might have been non-monotonous. Fig. 2 presents the calculated dependence for chromium-nickel-molybdenum steel. Material parameters have been determined on the basis of experimental data [8]. Domain below to the calculated curve corresponds to brittle fracture.

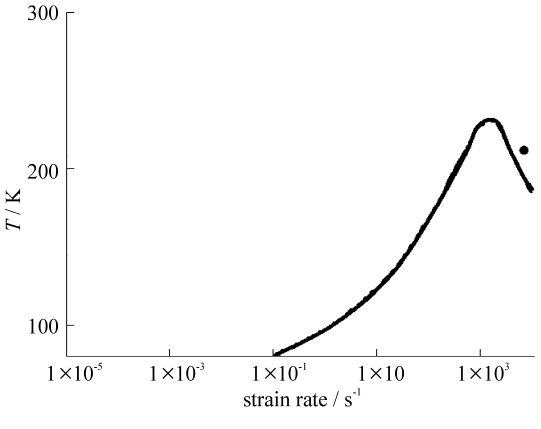


Fig. 2 Dependence of transition temperature on strain-rate for chromium-nickel-molybdenum steel [8]

3 Generalization of yield criterion

Dynamic yield criterion (2) can be considered in frame of more general approach based on the notion of “fading memory” [3]. According to this approach the current value of applied stress in quasi-static criterion (3) should be replaced by “relaxed” stress, i.e. by functional

$$I(t)=\int_0^t \sigma(s)K(t-s)\,\mathrm{d}s<\sigma_Y\ ,$$

where $K(t)$ is the function of fading memory. $K(t)$ is nonnegative non-increasing function satisfying the condition:

$$\int_0^{+\infty} K(t)\,\mathrm{d}t=1\,.$$

Criterion (2) is evidently obtained if

$$K(t)=\begin{cases}1/\tau, & 0\leqslant t\leqslant\tau,\\ 0, & \text{otherwise.}\end{cases}$$

So incubation time is the time of full fading which supposed to be finite. If we deal with simultaneous processes with different characteristic time it may be useful to replace single incubation time by relaxation spectrum. We define

$$I(t)=\int_0^{+\infty}\varphi(p)\left(\int_{t-\frac{1}{p}}^{t}\sigma(s)\cdot H(s)\cdot\mathrm{d}s\right)\mathrm{d}p\,,$$

where p is frequency, $\varphi(p)$ is spectral function satisfying condition

$$\int_0^{+\infty}\frac{\varphi(p)}{p}\,\mathrm{d}p=1\,.$$

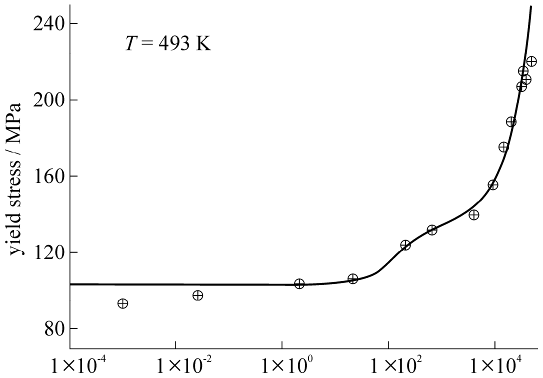
Incubation-time model corresponds to

$$\varphi(p) = \frac{\delta(p\tau - 1)}{\tau},$$

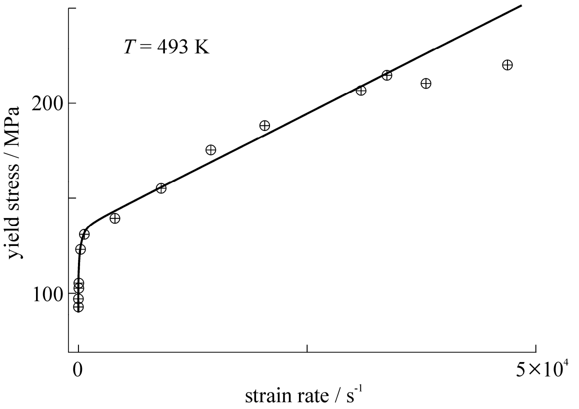
where $\delta(t)$ is Dirac delta function. Consider for example two-level model. Each level is defined by characteristic time τ_k and threshold level of stress σ_k. Then we obtain following criterion

$$\left(\frac{1}{\sigma_1} - \frac{1}{\sigma_2}\right) \cdot \frac{1}{\tau_1} \cdot \int_{t-\tau_1}^{t} \sigma(s)H(s)\mathrm{d}s + \frac{1}{\sigma_2} \cdot \frac{1}{\tau_2} \cdot \int_{t-\tau_2}^{t} \sigma(s)H(s)\mathrm{d}s < 1. \qquad (7)$$

The calculated curve for following values of parameters $\tau_1 = 4.5 \times 10^{-6}$ s, $\tau_2 = 2.4 \times 10^{-8}$ s, $\sigma_1 = 103$ MPa, $\sigma_2 = 135$ MPa compared on the Fig. 3 with experimental data for mild steel [9].



(a) semi-logarithmic scale



(b) linear scale

Fig. 3 Dependence of yield limit of mild-steel on temperature and strain rate model (Eq. 7) and experiment [9]

It should be noted that in case of multilevel model the shape parameter is not introduced ($\alpha = 1$). Some other variants of choice of spectral function $\varphi(p)$ have also been analyzed.

4 Conclusion

Incubation-time approach is applied to the modeling of yielding of metals and alloys. For given temperature only two additional parameters of material should be experimentally determined. Criterion is valid for wide range of loading rate; it allows using this criterion for the analysis of thermal effect and failure mode transition. Analysis shows that simplified models fail to explain behavior of real materials. Proposed temperature dependencies allow to include both “normal” thermal softening and “anomalous” increasing of yield stress. Similar approach has been used for the analysis of martensitic inelasticity in shape memory NiTi alloy [10].

References:

[1] Bratov V, Morozov N, Petrov Y. Dynamic Strength of Continuum[M]. Saint Petersburg: Saint Petersburg University Press, 2009.

[2] Kraft J M, Sullivan A M. Effect of grain size and carbon content on yield delay time of mild steel[J]. Trans of Am Soc Metals, 1959, 51:643-648.

[3] Rabotnov Yu N. Elements of hereditary solid mechanics [M]. Moscow: Mir Publishers, 1980.

[4] Gruzdkov A A, Petrov Y V. On temperature-time corresponddence in high-rate deformation of metals[J]. Doklady Physics, 1999, 44(2):114-116.

[5] Gruzdkov A A, Petrov Y V, Smirnov V I. An invariant form of the dynamic criterion for yield of metals[J]. Physics of the Solid State, 2002, 44(11):2080-2082.

[6] Gruzdkov A A, Sitnikova E V, Petrov Y V, et al. Thermal effect in dynamic yielding and fracture of metals and alloys[J]. Mathematics and Mechanics of Solids, 2009, 14 (1/2):72-87.

[7] Petrov Yu V, Gruzdkov A A, Sitnikova E V. Anomalous behavior of yield stress upon an increase in temperature under high strain rate conditions[J]. Doklady Physics. 2007, 52(12):691-694.

[8] Vashchenko A P, Leonov V P, Tokarev V M, et al. Effect of rapid deformation and temperature on the strength and plasticity characteristics of chromium-nickel-molybde-

num steel[J]. Strength of Materials, 1992, 23(9):951-954.

[9] Campbell J D, Ferguson W G. The temperature and strain-rate dependence of shear strength of mild steel[J]. The Philosophical Magazine, 1970, 21:63-68.

[10] Gruzdkov A, Krivosheev S, Petrov Yu, et al. Martensitic inelasticity of Ni-Ti shape-memory alloy under pulsed loading[J]. Materials Science and Engineering 2008, 481-482:105-108.

基于孕育时间的金属材料动态屈服理论

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摘要：利用孕育时间准则建立金属从弹性到塑性状态的转变模型，这种方法类似于在 St.-Petersburg State University 发展起来的脆性固体断裂的结构暂存理论．该方法可使我们建立模型，很好地分析和解释材料随温度变化发生的屈服应力演化、韧-脆性失效转变及其他效应等特殊的力学现象.

关键词：动态屈服；孕育时间方法；失效准则

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