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Swelling and Folding as Mechanisms of 3D Shape Formation in Thin Elastic Sheets

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Abstract

We work with two different mechanisms to generate geometric frustration on thin elastic sheets; isotropic differential growth and folding. We describe how controlled growth and prescribing folding patterns are useful tools for designing three-dimensional objects from information printed in two dimensions. The first mechanism is inspired by the possibility to control shapes by swelling polymer films, where we propose a solution for the problem of shape formation by asking the question, ``what 2D metric should be prescribed to achieve a given 3D shape?'', namely the reverse problem. We choose two different types of initial configurations of sheets, disk-like with one boundary and annular with two boundaries. We demonstrate our technique by choosing four examples of 3D axisymmetric shapes and finding the respective swelling factors to achieve the desired shape. Second, we present a mechanical model for a single curved fold that explains both the buckled shape of a closed fold and its mechanical stiffness. The buckling arises from the geometrical frustration between the prescribed crease angle and the bending energy of the sheet away from the crease. This frustration increases as the sheet's area increases. Stiff folds result in creases with constant space curvature while softer folds inherit the broken symmetry of the buckled shape. We extend the application of our numerical model to show the potential to study multiple fold structures.