國　立　清　華　大　學　命　題　紙

九十二學年度＿＿電子工程研判所＿＿系（所）＿＿＿＿＿＿組碩士班研究生招生考試

科目＿＿近代物理＿＿科號＿2604＿ 共＿1＿頁第＿1＿頁 *請在試卷【答案卷】內作答

1. A free particle of mass m moving in one dimension is known to be in the initial state

$$\psi(x,0) = \sin(k_o x).$$

(a) What is $\psi(x,t)$? (5%)

(b) What values of p (momentum) will measurement yield at time t, and with what probabilities will these values occur? (5%)

(c) Suppose that p is measured at $t = 5\,\mathrm{s}$ and the value $\hbar k_o$ is found. What is $\psi(x,t)$ at $t > 5\,\mathrm{s}$? (5%)

2. Consider a one-dimensional box problem. The width of the box is a. A particle is moving freely in the box. The eigenstates can be written as

$$\varphi_n = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right), \qquad n = 1,2,3,\ldots,$$

and the eigen-energies can be written as

$$E_n = n^2 E_1, \qquad E_1 = \frac{\hbar^2\pi^2}{2ma^2}, \qquad n = 1,2,3,\ldots..$$

Suppose the particle is initially in the state of the form

$$\psi(x,0) = \frac{2\varphi_2 + 3\varphi_3 + 5\varphi_5}{\sqrt{38}}.$$

(a) What is the wave function of the particle at time t? (5%)

(b) What is the probability of finding the eigen-energy E_3 at time t? (5%)

(c) Find the expectation energy of the particle at time t. (5%)

(d) Find the expectation momentum of the particle at time t. (5%)

3. Consider a two-dimensional **free particle** confined in a **square box**. Let m = particle mass, L = length of the box.

a) Derive its **ground state** energy and wave function. (10%)

b) Derive its **first excited state** energy and wave functions. (Note that the first excited states are degenerate, and you must find all of them.) (10%)

4. Please describe a quantum-mechanical phenomenon which **clearly** illustrates the uncertainty principle, $\Delta x \Delta p \geq \hbar$. (15%)

5. (a) Assuming that hydrogen atom has a circular electron orbit and the centripetal force and the electric force are balanced, calculate the total energy of the hydrogen atom using classical dynamics.
(b) Bohr's condition for hydrogen atom is that an electron orbit contains an integral number of de Broglie wavelengths. Find the orbital radius of hydrogen atom in this Bohr model.
Use symbols as follows: m = electron mass, e = electronic charge, E = energy, r = radius of electron orbit, F = force, v = velocity, ε_0 = permittivity in vacuum, h = Planck constant.
(15%)

6. (a) Given that the density of electron states g(E) as a function of energy E is $g(E) = (8\sqrt{2}\,\pi V m^{3/2}/h^3)E^{1/2}$, find the Fermi energy E_F of a metal with total number of free electrons N and volume V.
(b) Based on the result from part (a), calculate the Fermi energy E_F in copper on the assumption that each copper atom contributes one free electron to the electron gas. The density of copper is 8.94×10^3 kg/m^3 and its atomic mass is 63.5 atomic mass unit.
(1 atomic mass unit $=1.66\times10^{-27}$ kg, electron rest mass m $= 9.11\times10^{-31}$ kg, Planck constant h $= 6.626\times10^{-34}$ J-s.)
(15%)