類組：電機類　科目：近代物理(300F)　　※選擇題請在答案卡內作答，非選擇題請在答案卷內作答

一、單選題 (共計 6 題，每題 5 分，答錯 1 題倒扣 1.25 分)

1. Suppose in an accelerator, an electron is accelerated to the speed of 0.99999999967c (c = speed of light). What is the kinetic energy of the electron? (A) 3.2×10^{-8} J, (B) 3.2×10^{-9} J, (C) 2.2×10^{-8} J, (D) 2.2×10^{-9} J, (E) none of the above.

2. If a wave has the dispersion $w(k) = \hbar k^2/(2m)$, where w = angular frequency and k = wave vector. Then, what is the velocity of the wave $\psi(x,t) = \int_{k_c-\delta}^{k_c+\delta} dk \exp[i(kx - wt)]$? Here, $k_c < 0$, and $|k_c| >> \delta$.

   (A) $\hbar\delta/(2m)$ towards **+x** direction,

   (B) $\hbar|k_c|/(2m)$ towards **−x** direction,

   (C) $\hbar\delta/m$ towards **+x** direction,

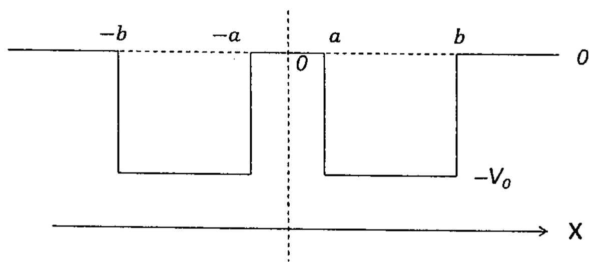
   (D) $\hbar|k_c|/m$ towards **−x** direction,

   (E) none of the above.

3. Here are two quantum wells in the regions $x \in [-b, -a]$ and $x \in [a, b]$. The wavefunction (φ) is expanded along the *x*-axis and satisfies the equations shown below. The $\hbar$ is the Dirac constant and *m* is the mass of the particle. The energy (*E*) of particle is higher than $-V_0$ and less than zero, as illustrated below.

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi(x) = (E + V_0)\varphi(x) \qquad -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi(x) = E\varphi(x)$$

$-b < x < -a;\quad a < x < b$　　　　*otherwise*



On this physical system, select the most suitable description from the following options:

(A) These two wells can be regarded as always independent on each other by the potential barrier in the region $x \in [-a, a]$.

(B) These two wells can be regarded as independent on each other by the potential barrier in the region $x \in [-a, a]$ if a is small enough even while V_0 is finite.

(C) These two wells can be combined to make coupled states by tunneling phenomena across the potential barrier in the region $x \in [-a, a]$; which is decomposed into symmetric state and anti-symmetric state, if a and V_0 are small enough.

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注意:背面有試題

(D) These two wells can be combined to make a unique mixing state by tunneling phenomena across the potential barrier in the region $x \in [-a, a]$, if a and V_0 are small enough.

(E) These two wells can be combined by tunneling phenomena across the potential barrier in the region $x \in [-a, a]$ if a and V_0 are small enough, but the eigenstates are independent.

4. On Schrodinger equation of a particle, select the most suitable description from the following options:
   (A) By solving the Schrodinger equation, we can tell where the particle existed but not when the particle existed there.
   (B) By solving the Schrodinger equation, we can tell where the particle will exist but not when the particle will exist there.
   (C) By solving the Schrodinger equation, we cannot tell where the particle existed but can tell when the particle existed.
   (D) By solving the Schrodinger equation, we cannot tell where the particle will exist but can tell when the particle will exist.
   (E) By solving the Schrodinger equation, we cannot tell when and where the particle exists.

5. Stern-Gerlach conducted an experiment in 1921 that showed
   (A) energy quantization.
   (B) orbital angular momentum quantization.
   (C) magnetic orbital quantization.
   (D) space quantization.
   (E) particle-wave duality.

6. Which of the following statements is true:
   (A) $\vec{L}$ can never be perpendicular to $\vec{B}$,
   (B) $\vec{L}$ can never be aligned parallel to $\vec{B}$,
   (C) $\vec{L}$ can be aligned parallel to $\vec{B}$,
   (D) $\vec{L}$ must be perpendicular to $\vec{B}$,
   (E) $\vec{L}$ does not response to $\vec{B}$,

   where $\vec{L}$ is the angular momentum of the orbital electron and $\vec{B}$ is the external magnetic field.

二、複選題 (共計 6 題，每題 5 分，答錯 1 題倒扣 1 分)

7. Which of the following statements are correct in regard to the specific heat of an insulator 'c'? Let N_A = Avogadro's number and k_B = Boltzmann constant.
   (A) The classical limit of 'c' = $3k_BN_A$ at low temperatures,

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(B) The classical limit of ‘c’ = $3k_BN_A$ at high temperatures,

(C) Due to energy quantization, ‘c’ approaches $3k_BN_A$ at high temperatures,

(D) Due to energy quantization, ‘c’ approaches zero at low temperatures,

(E) ‘c’ is temperature independent.

8. Which of the following statements are correct in regard to the Planck distribution $f(T)=\frac{1}{e^{\hbar w/k_BT}-1}$ for photons?

(A) f(T) = the average energy of EM (electromagnetic) radiation at angular frequency w,

(B) f(T) takes into account the energy quantization of EM radiation,

(C) f(T) for $\hbar w << k_BT$ is approximately linear in T,

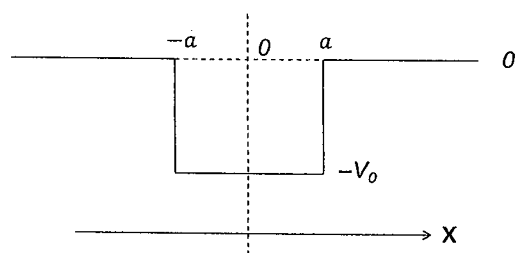
(D) Using f(T), the average EM energy at a mode of angular frequency w is approximately given by k_BT, for $\hbar w << k_BT$,

(E) f(T) shows strong quantum effects for $\hbar w >> k_BT$.

9. Here is one-dimensional potential well in the region $x \in [-a, a]$ with a particle whose energy (E) is higher than $-V_0$ and less than zero, as illustrated below. The wavefunction (φ) of this particle is expanded along the x-axis and satisfies the equations shown below. The $\hbar$ is the Dirac constant and m is the mass of the particle.

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi(x)=(E+V_0)\varphi(x) \quad -a<x<a$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi(x)=E\varphi(x) \quad \textit{otherwise}$$



On this physical system, select the suitable descriptions from the following options:

(A) The wavefunction is completely confined within the well forever as long as V_0 is finite.

(B) The wavefunction is confined around the well with tails out of the well to an extent as long as V_0 is finite.

(C) The particle staying in the well will emit to the right hand side ($x = \infty$) after the emission time has passed.

(D) The wavefunction is always symmetric around $x = 0$.

(E) The wavefunction is sometimes symmetric around $x = 0$ and at other times anti-symmetric around $x = 0$.

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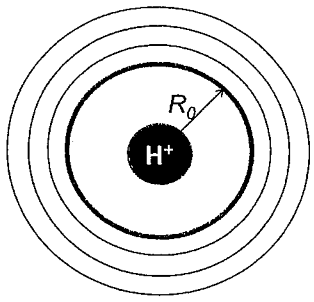
10. Here is a simple model of hydrogen atom. An electron can exist in orbitals surrounding hydrogen ion (H^{+}) with the minimum radius being R_0, as illustrated below. In this simple model, electron's orbital is circles on sheet. The electron on orbitals can exchange energy with light to hop between orbitals.



Select the suitable descriptions from the following options:

(A) $2\pi R_0$ is a wavelength of the electron on the orbital with the radius being R_0.

(B) R_0 is a wavelength of the electron on the orbital with the radius being R_0.

(C) An electron existing inside the circle with the radius being R_0 will collapse with H^+; and then will cause the hydrogen explosion.

(D) Heisenberg's Uncertainty Relation prohibits any electron to exist inside the circle with the radius being R_0.

(E) Heisenberg's Uncertainty Relation cannot prohibit an electron to exist inside the circle with the radius being R_0 for a certain limited period of time.

11. Which of the following statements are true:

(A) The conductivity of semiconductors increases with increasing temperature. The number of conduction electrons and the number of holes increase steeply with increasing temperature.

(B) When two metals with different work functions are put together, they will have the same potential because of the intrinsic property of metals.

(C) There is no conduction band in insulators because they cannot conduct electricity.

(D) The energy gap of a semiconductor decreases with increasing temperature.

(E) The conductivity of metals decreases with increasing temperature due to electron collisions with vibrating atoms.

12. Which of the following statements are true:

(A) The Fermi energy corresponds to the maximum energy electrons can have in a metal at T = 0K.

(B) If an electric field is applied to a metal, electrons having energy near the Fermi energy require only a small amount of additional energy from the applied field to reach nearby empty energy states.

(C) When the particle concentration is low, the particle mass is high and temperature is high, Fermi-Dirac distribution can be approximated by Maxwell-Boltzmann distribution.

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參考用

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(D) The average kinetic energy of electrons in a metal at T = 0K is zero.

(E) The reason why electrons in a metal can be described by Fermi-Dirac distribution is because electrons are identical, indistinguishable and Coulomb repulsive to each other.

三、非選擇題 (共計 3 大題)

1. (a) (4%) Estimate the number of photons emitted per second from a 60 W incandescent bulb assuming the total efficiency between (visible) light and electricity is 3%; use 575 nm as the average wavelength of the (visible) light emitted. Is the quantum nature of this radiation important?

   (b) (9%) A 0.7 MeV photon scatters from an electron initially at rest. If the photon scatters at an angle of 35°, calculate the energy and wavelength of the scattered photon; the kinetic energy of the recoiling electron; and the angle at which the electron recoils.

   (c) (7%) Hydrogen atom in its ground state is excited by means of a monochromatic radiation of wavelength 97.06 nm. How many different wavelengths are possible in the resulting emission spectrum? Find the longest wavelength among these.

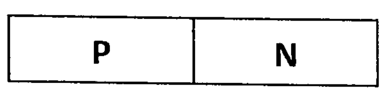
2. (10%) Hydrogen atom model:

   (a) (5%) **Verify that the average value of 1/r for 1s electron in the hydrogen atom is $1/a_0$.** The wave function of a 1s electron is

$$\psi = \frac{e^{-r/a_0}}{\sqrt{\pi}\cdot a_0^{3/2}} \qquad \text{Useful formula: } \int xe^{bx}dx = \frac{e^{bx}}{b}\left(x-\frac{1}{b}\right)$$

   (b) (5%) **Determine the probability ratio** $P_{a_0}/P_{a_0/2}$, that the 1s electron in a hydrogen atom to be at the distance a_0, relative to the 1s electron at the distance $a_0/2$. The radial function is $R=\frac{2}{a_0^{3/2}}e^{-r/a_0}$.

3. (10%) PN semiconductor junction diode as following figure:



   (a) (5%) Describe the carriers flow in junction region right after PN contacting. Plot the energy band diagram of PN junction in equilibrium. You must indicate the E_c (conduction band energy), E_v (valence band energy), and E_f (Fermi energy).

   (b) (5%) Illustrate the operation principle of light-emitting diode (LED).