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# A Space-time Smooth Artificial Viscosity Method For Nonlinear Conservation Laws

Jon Reisner, Jonathan Serencsa, Steve Shkoller

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We introduce a new methodology for adding localized, space-time smooth, artificial viscosity to nonlinear systems of conservation laws which propagate shock waves, rarefactions, and contact discontinuities, which we call the $C$-method. We shall focus our attention on the compressible Euler equations in one space dimension. The novel feature of our approach involves the coupling of a linear scalar reaction-diffusion equation to our system of conservation laws, whose solution $C(x,t)$ is the coefficient to an additional (and artificial) term added to the flux, which determines the location, localization, and strength of the artificial viscosity. Near shock discontinuities, $C(x,t)$ is large and localized, and transitions smoothly in space-time to zero away from discontinuities. Our approach is a provably convergent, spacetime-regularized variant of the original idea of Richtmeyer and Von Neumann, and is provided at the level of the PDE, thus allowing a host of numerical discretization schemes to be employed. We demonstrate the effectiveness of the $C$-method with three different numerical implementations and apply these to a collection of classical problems: the Sod shock-tube, the Osher-Shu shock-tube, the Woodward-Colella blast wave and the Leblanc shock-tube. First, we use a classical continuous finite-element implementation using second-order discretization in both space and time, FEM-C. Second, we use a simplified WENO scheme within our $C$-method framework, WENO-C. Third, we use WENO with the Lax-Friedrichs flux together with the $C$-equation, and call this WENO-LF-C. All three schemes yield higher-order discretization strategies, which provide sharp shock resolution with minimal overshoot and noise, and compare well with higher-order WENO schemes that employ approximate Riemann solvers, outperforming them for the difficult Leblanc shock tube experiment.

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## Submission history