1.

(a) Write down the differential forms of Maxwell's equations. (5%)

(b) State Helmholtz's theorem in words. (5%)

(c) If a small single-turn loop antenna has a radiation resistance of 0.01 Ω, how many turns are needed to give a radiation resistance of 1 Ω? (5%)

(d) A thin wire antenna of length 0.01 $\lambda$ has a radiation resistance of 0.08 Ω. What is the radiation resistance for such an antenna, if the length is scaled to 0.05 $\lambda$? (5%)

(e) Find the magnetic field $\mathbf{H}$ at the center of a square loop carrying a current $I$. The side of this square loop is $\ell$ meters. (5%)

2. In a time-varying situation, how do we define a good conductor? Seawater can be characterized by $\sigma = 4\mho/\text{m}$, $\epsilon = 81\epsilon_0$, and $\mu = \mu_0$. Can seawater be considered as a good conductor at $f = 1$ MHz. (10%)

3. The definition of the curl of a vector field $\mathbf{B}$, written as $\nabla \times \mathbf{B}$, is a vector which is directed in the normal direction of the area, defined by the familiar right-hand rule, while the area is oriented to make the following circulation over the peripheral of the area maximum:

$$\nabla \times \mathbf{B} = \lim_{\Delta S \to 0} \frac{1}{\Delta S}\left[\hat{n} \oint \mathbf{B} \cdot d\mathbf{l}\right],$$

where $\hat{n}$ is the outward normal unit vector. Consider generalized orthogonal coordinates. Prove

$$\nabla \times \mathbf{B} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} \hat{u}_1 h_1 & \hat{u}_2 h_2 & \hat{u}_3 h_3 \\ \dfrac{\partial}{\partial u_1} & \dfrac{\partial}{\partial u_2} & \dfrac{\partial}{\partial u_3} \\ h_1 B_1 & h_2 B_2 & h_3 B_3 \end{vmatrix},$$

where $h_1$, $h_2$, and $h_3$ are the metric coefficients of the respective coordinates. You will be given zero credit if you change the present coordinates into the Cartesian coordinates. (15%)

4. Along the signal propagation direction, the equivalent circuit of a lossless two-conductor transmission line can be treated as inductors in series. Explain clearly the physics behind the mentioned model, i.e., the reason of the presence of inductance. (10%)

5. Consider a plane lightwave obliquely incident toward the interfaces of two flat, dielectric media. The reflection of the wave is drastic different between the TE- and the TM-polarized light. Which case can show a zero reflection? Explain clearly the reasons in your answer. (10%)

6. Given six air-filled rectangular waveguides of the following inner dimensions:
   (a) 40 cm×40 cm, (b) 20 cm×10 cm, (c) 4 cm×4 cm,
   (d) 2 cm×1 cm, (e) 0.4 cm×0.4 cm, and (f) 0.2 cm×0.1 cm.
   Determine in which waveguide(s) the $TE_{10}$ mode can propagate and in which waveguide(s) only the $TE_{10}$ mode can propagate. The operating frequency $f = 10$ GHz. (10%)

7. In a source-free dielectric material of inhomogeneous permittivity $\epsilon(\mathbf{r})$, the divergence of the electric field $\mathbf{E}$ is not equal to zero but to a function involving the electric field $\mathbf{E}$ itself. Show that the relation is given as

$$\nabla \cdot \mathbf{E}(\mathbf{r}) = \mathbf{g}(\mathbf{r}) \cdot \mathbf{E}(\mathbf{r}),$$

   and find the vector function $\mathbf{g}(\mathbf{r})$. (10%)

8. The Lorentz force law describes the electromagnetic force exerted on a particle of charge $q$ and velocity $\mathbf{v}$. This force can be expressed in several ways. Write down the Lorentz force law in terms of
   (a) electric field $\mathbf{E}(\mathbf{r}, t)$ and magnetic field $\mathbf{B}(\mathbf{r}, t)$,
   (b) electric scalar potential $\Phi(\mathbf{r}, t)$ and magnetic vector potential $\mathbf{A}(\mathbf{r}, t)$,
   (c) charge density $\rho_n(\mathbf{r}, t)$ and current density $\mathbf{J}(\mathbf{r}, t)$.
   The time retardation should be considered and the arguments of each function should be given clearly. (10%)

**Constants for Reference**

$\epsilon_0 \simeq \frac{1}{36\pi} \times 10^{-9}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ H/m