

## Mathematical Physics

# Caustics, counting maps and semi-classical asymptotics

N. M. Ercolani

*(Submitted on 10 Dec 2009)*

This paper develops a deeper understanding of the structure and combinatorial significance of the partition function for Hermitean random matrices. The coefficients of the large  $N$  expansion of the logarithm of this partition function, also known as the genus expansion, (and its derivatives) are generating functions for a variety of graphical enumeration problems. The main results are to prove that these generating functions are in fact specific rational functions of a distinguished irrational (algebraic) function of the generating function parameters. This distinguished function is itself the generating function for the Catalan numbers (or generalized Catalan numbers, depending on the choice of parameter). It is also a solution of the inviscid Burgers equation for certain initial data. The shock formation, or caustic, of the Burgers characteristic solution is directly related to the poles of the rational forms of the generating functions.

These results in turn provide new information about the asymptotics of recurrence coefficients for orthogonal polynomials with respect to exponential weights. One gains new insights into the relation between certain derivatives of the genus expansion and the asymptotic expansion of the first Painleve transcendent, related to the double-scaling limit. This work provides a precise expression of the Painleve asymptotic coefficients directly in terms of the coefficients of the partial fractions expansion of the rational form of the generating functions established here. Moreover, these insights point toward a more general program relating the first Painleve hierarchy and the higher order structure of the double-scaling limit to the specific rational structure of generating functions.

Subjects: **Mathematical Physics (math-ph)**; Combinatorics (math.CO); Exactly Solvable and Integrable Systems (nlin.SI)

Cite as: [arXiv:0912.1904v1](#) [math-ph]

## Submission history

From: Nicholas Michael Ercolani [[view email](#)]  
[v1] Thu, 10 Dec 2009 01:52:32 GMT (33kb)

*[Which authors of this paper are endorsers?](#)*

## Download:

- [PDF](#)
- [PostScript](#)
- [Other formats](#)

Current browse context:

**math-ph**

[< prev](#) | [next >](#)

[new](#) | [recent](#) | [0912](#)

Change to browse by:

[math](#)

[math.CO](#)

[nlin](#)

[nlin.SI](#)

## References & Citations

- [CiteBase](#)

## Bookmark([what is this?](#))

- [CiteULike logo](#)
- [Connotea logo](#)
- [BibSonomy logo](#)
- [Mendeley logo](#)
- [Facebook logo](#)
- [del.icio.us logo](#)
- [Digg logo](#)
- [Reddit logo](#)