High Energy Physics - Theory

Geometry and field theory in multi-fractional spacetime

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We construct a theory of fields living on continuous geometries with fractional Hausdorff and spectral dimensions, focussing on a flat background analogous to Minkowski spacetime. After reviewing the properties of fractional spaces with fixed dimension, presented in a companion paper, we generalize to a multi-fractional scenario inspired by multi-fractal geometry, where the dimension changes with the scale. This is related to the renormalization group properties of fractional field theories, illustrated by the example of a scalar field. Depending on the symmetries of the Lagrangian, one can define two models. In one of them, the effective dimension flows from 2 in the ultraviolet (UV) and geometry constrains the infrared limit to be four-dimensional. At the UV critical value, the model is rendered power-counting renormalizable. However, this is not the most fundamental regime. Compelling arguments of fractal geometry require an extension of the fractional action measure to complex order. In doing so, we obtain a hierarchy of scales characterizing different geometric regimes. At very small scales, discrete symmetries emerge and the notion of a continuous spacetime begins to blur, until one reaches a fundamental scale and an ultra-microscopic fractal structure. This fine hierarchy of geometries has implications for non-commutative theories and discrete quantum gravity. In the latter case, the present model can be viewed as a top-down realization of a quantum-discrete to classical-continuum transition.

Comments:	1+82 pages, 1 figure, 2 tables. v2-3: discussions clarified and improved (especially section 4.5), typos corrected, references added; v4: further typos corrected
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