Mathematical Physics

An effective medium approach to the asymptotics of the statistical moments of the parabolic Anderson model and Lifshitz tails

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Originally introduced in solid state physics to model amorphous materials and alloys exhibiting disorder induced metal-insulator transitions, the Anderson model $H_{\omega}= -\Delta + V_{\omega} $ on $l^2(\bZ^d)$ has become in mathematical physics as well as in probability theory a paradigmatic example for the relevance of disorder effects. Here Δ is the discrete Laplacian and $V_{\omega} = \{V_{\omega}(x): x \in \bZ^d\}$ is an i.i.d. random field taking values in $\bR$. A popular model in probability theory is the parabolic Anderson model (PAM), i.e. the discrete diffusion equation $\partial_t u(x,t) =-H_{\omega} u(x,t)$ on $ \bZ^d \times \bR_+$, $u(x,0)=1$, where random sources and sinks are modelled by the Anderson Hamiltonian. A characteristic property of the solutions of (PAM) is the occurrence of intermittency peaks in the large time limit. These intermittency peaks determine the thermodynamic observables extensively studied in the probabilistic literature using path integral methods and the theory of large deviations. The rigorous study of the relation between the probabilistic approach to the parabolic Anderson model and the spectral theory of Anderson localization is at least mathematically less developed. We see our publication as a step in this direction. In particular we will prove an unified approach to the transition of the statistical moments $< u (0,t) >$ and the integrated density of states from classical to quantum regime using an effective medium approach. As a by-product we will obtain a logarithmic correction in the traditional Lifshitz tail setting when V_{ω} satisfies a fat tail condition.

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