Mathematical Physics

Higher Symplectic Geometry

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(Submitted on 21 Jun 2011)

We consider generalizations of symplectic manifolds called n-plectic manifolds. A manifold is n-plectic if it is equipped with a closed, nondegenerate form of degree n+1. We show that higher structures arise on these manifolds which can be understood as the categorified or homotopy analogues of important structures studied in symplectic geometry and geometric quantization. Just as a symplectic manifold gives a Poisson algebra of functions, we show that any n-plectic manifold gives a Lie n-algebra containing certain differential forms which we call Hamiltonian. Lie n-algebras are examples of strongly homotopy Lie algebras. They consist of an n-term chain complex equipped with a collection of skew-symmetric multi-brackets that satisfy a generalized Jacobi identity. We then develop the machinery necessary to geometrically quantize n-plectic manifolds. In particular, just as a prequantized symplectic manifold is equipped with a principal U(1)-bundle with connection, a prequantized 2-plectic manifold is equipped with a U(1)-gerbe with 2-connection. A gerbe is a categorified sheaf, or stack, which generalizes the notion of a principal bundle. Furthermore, over any 2-plectic manifold there is a vector bundle equipped with extra structure called a Courant algebroid. This bundle is the 2-plectic analogue of the Atiyah algebroid over a prequantized symplectic manifold. Its space of global sections also forms a Lie 2-algebra, which we use to prequantize the Lie 2-algebra of Hamiltonian forms. Finally, we introduce the 2-plectic analogue of the Bohr-Sommerfeld variety associated to a real polarization, and use this to geometrically quantize 2-plectic manifolds. The output of this procedure is a category of quantum states. We consider a particular example in which the objects of this category can be identified with representations of the Lie group SU(2).

Comments:	Ph.D. thesis, 160 pages. Department of Mathematics, University of California, Riverside, 2011
Subjects:	**Mathematical Physics (math-ph)**; Differential Geometry (math.DG)
Cite as:	**arXiv:1106.4068 [math-ph]**
	(or **arXiv:1106.4068v1 [math-ph]** for this version)

Submission history

From: Christopher Rogers [view email]
[v1] Tue, 21 Jun 2011 01:26:18 GMT (123kb)