Mathematical Physics

Snub 24-Cell Derived from the Coxeter-Weyl Group W(D4)

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Snub 24-cell is the unique uniform chiral polytope in four dimensions consisting of 24 icosahedral and 120 tetrahedral cells. The vertices of the 4-dimensional semi-regular polytope snub 24-cell and its symmetry group ${(W(D_{4})\mathord{/{\vphantom {(W(D_{4}) C_{2}}}. \kern-\nulldelimiterspace} C_{2}}):S_{3} $ of order 576 are obtained from the quaternionic representation of the Coxeter-Weyl group \textbf{$W(D_{4}).$}The symmetry group is an extension of the proper subgroup of the Coxeter-Weyl group \textbf{$W(D_{4})$}by the permutation symmetry of the Coxeter-Dynkin diagram \textbf{$D_{4} .$} The 96 vertices of the snub 24-cell are obtained as the orbit of the group when it acts on the vector \textbf{$\Lambda =(\tau, 1, \tau, \tau)$}or\textbf{}on the vector\textbf{$\Lambda = (\sigma, 1, \sigma, \sigma)$}in the Dynkin basis with\textbf{$\tau =\frac{1+\sqrt{5}}{2} {\rm and}\sigma =\frac{1-\sqrt{5}}{2} {\rm .}$} The two different sets represent the mirror images of the snub 24-cell. When two mirror images are combined it leads to a quasi regular 4D polytope invariant under the Coxeter-Weyl group \textbf{$W(F_{4}).$}Each vertex of the new polytope is shared by one cube and three truncated octahedra. Dual of the snub 24 cell is also constructed. Relevance of these structures to the Coxeter groups \textbf{$W(H_{4}){\rm and}W(E_{8})$}has been pointed out.

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