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Hermitian Operators and Convex Functions

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Abstract: We establish several convexity results for Hermitian matrices. For instance: Let A, B be Hermitian and let f be a convex function. If X and Y stand for $f(\{A+B\}/2)$ and $\{f(A)+f(B)\}/2$ respectively, then there exist unitaries U, V such that

$$X \leq \frac{UYU^* + VYV^*}{2}.$$

Consequently, $\lambda_{2j-1}(X) \leq \lambda_j(Y)$, where $\lambda_j(\cdot)$ are the eigenvalues arranged in decreasing order.