An Overpartition Analogue of Bressoud's Theorem of Rogers-Ramanujan-Gordon Type

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Abstract: For $k \geqslant 2$ and $k \geqslant i \geqslant 1$, let $B_{k,i}(n)$ denote the number of partitions of n such that part 1 appears at most i-1 times, two consecutive integers l and $l+1$ appear at most k-1 times and if l and $l+1$ appear exactly k-1 times then the sum of the parts l and $l+1$ is congruent to i-1 modulo 2. Let $A_{k,i}(n)$ denote the number of partitions with parts not congruent to i, $2k$-i and $2k$ modulo $2k$. Bressoud's theorem states that $A_{k,i}(n) = B_{k,i}(n)$. Corteel, Lovejoy, and Mallet found an overpartition analogue of Bressoud's theorem for $i=1$, that is, for partitions not containing non-overlined part 1. We obtain an overpartition analogue of Bressoud's theorem in the general case. For $k \geqslant 2$ and $k \geqslant i \geqslant 1$, let $D_{k,i}(n)$ denote the number of overpartitions of n such that the non-overlined part 1 appears at most i-1 times, for any integer l, l and non-overlined $l+1$ appear at most k-1 times and if the parts l and the non-overlined part $l+1$ together appear exactly k-1 times then the sum of the parts l and non-overlined part $l+1$ has the same parity as the number of overlined parts that are less than $l + 1$ plus $i - 1$. Let $C_{k, i}(n)$ denote the number of overpartitions of n with the non-overlined parts not congruent to $\pm i$ and $2k - 1$ modulo $2k - 1$. We show that $C_{k, i}(n) = D_{k, i}(n)$. Note that this relation can also be considered as a Rogers-Ramanujan-Gordon type theorem for overpartitions.

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