

# The Rogers-Ramanujan-Gordon Theorem for Overpartitions

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**Abstract:** Let  $B_{k,i}(n)$  be the number of partitions of  $n$  with certain difference condition and let  $A_{k,i}(n)$  be the number of partitions of  $n$  with certain congruence condition. The Rogers-Ramanujan-Gordon theorem states that  $B_{k,i}(n) = A_{k,i}(n)$ . Lovejoy obtained an overpartition analogue of the Rogers-Ramanujan-Gordon theorem for the cases  $i = 1$  and  $i = k$ . We find an overpartition analogue of the Rogers-Ramanujan-Gordon theorem in the general case. Let  $D_{k,i}(n)$  be the number of overpartitions of  $n$  satisfying certain difference condition and  $C_{k,i}(n)$  be the number of overpartitions of  $n$  whose non-overlined parts satisfy certain congruences condition. We show that  $C_{k,i}(n) = D_{k,i}(n)$ . By using a function introduced by Andrews, we obtain a recurrence relation which implies that the generating function of  $D_{k,i}(n)$  equals the generating function of  $C_{k,i}(n)$ . We also find a generating function formula of  $D_{k,i}(n)$  by using Gordon marking representations of overpartitions, which can be considered as an overpartition analogue of an identity of Andrews for ordinary partitions.

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**Keywords:** overpartition, the Rogers-Ramanujan-Gordon theorem, the Gordon marking of an overpartition

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