

# The Extended Zeilberger Algorithm with Parameters

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**Abstract:** Two hypergeometric terms  $f(k)$  and  $g(k)$  are said to be similar if the ratio  $f(k)/g(k)$  is a rational function of  $k$ . For similar hypergeometric terms  $f_1(k), \dots, f_m(k)$ , we present an algorithm, called the extended Zeilberger algorithm, to derive a linear relation among the sums  $F_i = \sum_k f_i(k)$  ( $1 \leq i \leq m$ ) with polynomial coefficients. When the summands  $f_1(k), \dots, f_m(k)$  contain a parameter  $x$ , we further impose the condition that the coefficients of  $F_i$  in the linear relation are  $x$ -free. Such linear relations with  $x$ -free coefficients can be used to determine the structure relations for orthogonal polynomials and to derive recurrence relations for the connection coefficients between two sequences of orthogonal polynomials. The extended Zeilberger algorithm can be easily adapted to basic hypergeometric terms. As examples, we use the algorithm or its  $q$ -analogue to establish linear relations among orthogonal polynomials and to derive recurrence relations with multiple parameters for hypergeometric sums and basic hypergeometric sums.

**AMS Classification:** 33F10, 33C45, 33D45

**Keywords:** Zeilberger's algorithm, the extended Zeilberger algorithm, the Gosper algorithm, hypergeometric series, orthogonal polynomials

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