

Mixed Statistics on 01-Fillings of Moon Polyominoes

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Abstract: We establish a stronger symmetry between the numbers of northeast and southeast chains in the context of 01-fillings of moon polyominoes. Let M be a moon polyomino with n rows and m columns. Consider all the 01-fillings of M in which every row has at most one 1. We introduce four mixed statistics with respect to a bipartition of rows or columns of M . More precisely, let $S \subseteq \{1, 2, \dots, n\}$ and $R(S)$ be the union of rows whose indices are in S . For any filling M , the top-mixed (resp. bottom-mixed) statistic $\alpha(S;M)$ (resp. $\beta(S;M)$) is the sum of the number of northeast chains whose top (resp. bottom) cell is in $R(S)$, together with the number of southeast chains whose top (resp. bottom) cell is in the complement of $R(S)$. Similarly, we define the left-mixed and right-mixed statistics $\gamma(T;M)$ and $\delta(T;M)$, where T is a subset of the column index set $\{1, 2, \dots, m\}$. Let $\lambda(A;M)$ be any of these four statistics $\alpha(S;M)$, $\beta(S;M)$, $\gamma(T;M)$ and $\delta(T;M)$, we show that the joint distribution of the pair $(\lambda(A;M), \lambda(\bar{A};M))$ is symmetric and independent of the subsets S, T . In particular, the pair of statistics $(\lambda(A;M), \lambda(\bar{A};M))$ is equidistributed with $(\text{se}(M); \text{ne}(M))$, where $\text{se}(M)$ and $\text{ne}(M)$ are the numbers of southeast chains and northeast chains of M , respectively.

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