On the Combinatorics of the Boros-Moll Polynomials

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Abstract: The Boros-Moll polynomials arise in the evaluation of a quartic integral. The original double summation formula does not imply the fact that the coefficients of these polynomials are positive. Boros and Moll proved the positivity by using Ramanujan's Master Theorem to reduce the double sum to a single sum. Based on the structure of reluctant functions introduced by Mullin and Rota along with an extension of Foata's bijection between Meixner endofunctions and bi-colored permutations, we find a combinatorial proof of the positivity. In fact, from our combinatorial argument one sees that it is essentially the binomial theorem that makes it possible to reduce the double sum to a single sum.

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