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最大度为8且无4-扇的平面图的9-全可染性

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On the Total 9-colorability of Plane Graphs with Maximum Degree 8 without 4-fans

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摘要 设$G=(V, E)$是一个以V为顶点集，E为边集的图. 图G的一个k-全染色是一个映射φ：$V\cup E\to\{1, 2, \cdots, k\}$使得 $\varphi(x)\neq\varphi(y)$ 对所有相邻或相关联的元素x和y都成立. 若G有一个k-全染色，则说G是k-全可染的. 令Δ为G的最大度. 显然，对G进行全染色，至少需要Δ+1个颜色. Behzad和Vizing相互独立地猜想每个(简单)图都是(Δ+2)-全可染的. 已知最大度$\Delta\geqslant 9$的平面图是 (Δ+1)-全可染的. 通过研究极小反例的新的可约性质，本文运用权转移方法证明了最大度为8且不含4-扇的平面图是9-全可染的，这里的4-扇是指交于一点的4个相继的3-面. 这一结果改进了若干同类型的相关结果.

关键词： 平面图 全染色 最大度 扇

Abstract： Let $G=(V,E)$ be a graph with sets of vertices and edges V and E, respectively. A total k-coloring of G is a mapping φ: $V\cup E\to\{1, 2, \cdots, k\}$ such that $\varphi(x)\neq\varphi(y)$ whenever x and y are two adjacent or incident elements of $V\cup E$. G is totally k-colorable if it admits a total k-coloring. Let Δ denote the maximum degree of G. Clearly, at least Δ+1 colors are needed to totally color G. Behzad and Vizing independently conjectured that every graph is totally (Δ+2)-colorable. It is known that plane graphs with maximum degree $\Delta\geqslant$q9 are totally (Δ+1)-colorable. By exploring new reducibility of minimal counterexample, we use discharging method to prove that plane graphs with maximum degree 8 and without 4-fans are totally 9-colorable, where a k-fan in a plane graph consists of k consecutive 3-faces intersecting at a vertex. This improves some known results on the topic of total 9-colorability of plane graphs with maximum degree 8.

Key words： Plane graphs total coloring maximum degree fan

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