

## 非线性Sine-Gordon方程Hermite型有限元新的超收敛分析及外推

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## A New Superconvergence Analysis and Extrapolation of Hermite-type Finite Element for Nonlinear Sine-Gordon Equations

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**摘要** 在半离散格式下讨论了一类非线性Sine-Gordon方程的Hermite型矩形元逼近. 利用该元的高精度分析和对时间 $t$ 的导数转移技巧, 得到了 $H^1$ 模意义下 $O(h^2)$ 阶的最优误差估计和 $O(h^3)$ 阶的超逼近性. 进一步地, 通过运用插值后处理方法, 给出了超收敛结果. 与此同时, 借助于构造一个新的外推格式, 导出了与线性情形相同的 $O(h^4)$ 阶外推解.

**关键词:** Sine-Gordon方程 Hermite型矩形元 超逼近和超收敛 外推

**Abstract:** An Hermite-type rectangular element approximation is discussed for a class of nonlinear Sine-Gordon equations under semi-discrete scheme. The optimal error estimate with order  $O(h^2)$  and the superclose property with order  $O(h^3)$  in  $H^1$  norm are derived by use of high accuracy analysis of the element and the derivative transferring technique with respect to the time  $t$ . Moreover, the superconvergence result is obtained by the interpolation post-processing method. At the same time, the extrapolation solution with order  $O(h^4)$  is deduced through constructing a new extrapolation scheme, which is as same as that of the linear case.

**Key words:** Sine-Gordon equations Hermite-type rectangular element superclose and superconvergence extrapolation

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