

## 一维粘性液体-气体两相流模型自由边值问题全局强解存在唯一性

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## Existence and Uniqueness of Global Strong Solutions for One-dimensional Viscous Two-phase Model with Free Boundary Value Problem

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**摘要** 本文建立了一类粘性两相流模型, 主要研究了当初始密度间断连接到真空时的全局强解存在唯一性. 利用一系列的先验估计得到 $m$ 和 $n$ 的正上下界估计; 再运用差分方法, 证明了可压缩粘性液体-气体两相流模型的全局强解存在唯一性, 这样我们把Evje, Karlsen和姚磊, 朱长江的结论推广到 $\beta > 0$ ,  $\gamma > \max\{\beta + 1, 2\beta\}$ 的情形.

**关键词:** [两相流模型](#) [强解](#) [存在性](#) [唯一性](#)

**Abstract:** The purpose of this paper is to establish existence and uniqueness of strong solutions for a viscous of two-phase flow model when both the initial liquid and gas masses connect to vacuum discontinuously. This model is a drift-flux which is composed of two continuity equations and one mixture momentum equation supplemented. The liquid phase is assumed to be incompressible whereas the gas is described by a polytropic equation of state. The two-phase flow model can be reformulated in single-phase Navier-Stokes equations when we introduce an appropriate variable transformation. Applying techniques in studying Navier-Stokes equations and using a priori estimates, we get the positive upper and lower bound of  $m$  and  $n$ . Using the difference method to prove the existence and uniqueness of the global strong solutions. This improves the previous result of Evje, Karlsen and Yao, Zhu by enlarging the interval of  $\beta$  to  $\beta > 0$ ,  $\gamma > \max\{\beta + 1, 2\beta\}$ .

**Key words:** [two-phase flow model](#) [strong solution](#) [existence](#) [uniqueness](#)

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