

## 带Hardy项的半线性椭圆方程非球对称解的研究

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## Study on the Nonradial Solutions for Semilinear Elliptic Equation with Hardy Term

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摘要 本文考虑如下带Hardy项的半线性椭圆问题

$$\begin{cases} -\Delta u - \mu \frac{u}{|x|^2} = f(u), & x \in \Omega, \\ u = 0, & x \in \partial\Omega \end{cases}$$

非球对称解的存在性. 这里 $\Omega = \{x \in \mathbb{R}^n, n \geq 3, a < |x| < 1\}$  是  $\mathbb{R}^n$  ( $n \geq 3$ ) 中的环, 其中  $0 \leq \mu < \mu_c = ((n-2)/2)^2$ ,  $f(u)$  为已知函数. 本文在讨论球对称解的性质的基础上, 利用变分方法得到了方程的极小能量解的存在性, 并且利用分支理论得到了方程的非球对称解.

关键词: [非球对称解](#) [变分方法](#) [能量极小解](#) [分支理论](#)

Abstract: In this paper, we are concerned with the existence of positive radial and nonradial symmetric solutions for the following semilinear elliptic problem with Hardy term:

$$\begin{cases} -\Delta u - \mu \frac{u}{|x|^2} = f(u), & x \in \Omega, \\ u = 0, & x \in \partial\Omega \end{cases}$$

where  $\Omega = \{x \in \mathbb{R}^n, n \geq 3, a < |x| < 1\}$  is a annulus, and  $0 \leq \mu < \mu_c = ((n-2)/2)^2$ ,  $f(u)$  is some given function. Firstly, we discuss the detailed properties concerning the radial solutions and secondly we shall obtain the minimizing solutions by variational method. Lastly in section 4, we study the non-radial solution problem by bifurcation theory.

Key words: [nonradial symmetric solution](#) [minimizing solution](#) [variational method](#) [bifurcation theory](#)

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