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A new approach to the results of Kövari, Sós, and Turán concerning rectangle-free subsets of the grid

Jeremy F. Alm, Jacob Manske

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For positive integers \$m\$ and \$n\$, define \$f(m,n)\$ to be the smallest integer such that any subset \$A\$ of the \$m \times n\$ integer grid with \$|A| \geq f(m,n)\$ contains a rectangle; that is, there are \$x\in [m]\$ and \$y \in [n]\$ and \$d_{1},d_{2} \in \mathbb{Z}^{+}\$ such that all four points \$(x,y)\$, \$(x+d_{1},y)\$, \$(x,y+d_{2})\$, and \$(x+d_{1},y+d_{2})\$ are contained in \$A\$. In \cite{kovarisosturan}, K\"ovari, S\'os, and Tur\'an showed that \$\dlim_{k \to \infty}\dfrac{f(k,k)}{k^{3/2}} = 1\$. They also showed that whenever \$p\$ is a prime number, \$f(p^{2},p^{2}+p) = p^{2}(p+1)+1\$. We recover their asymptotic result and strengthen the second, providing cleaner proofs which exploit a connection to projective planes, first noticed by Mendelsohn in \cite{mendelsohn87}. We also provide an explicit lower bound for \$f(k,k)\$ which holds for all \$k\$.

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