



Mathematics > Metric Geometry

# Amenability, locally finite spaces, and bi-lipschitz embeddings

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We define the isoperimetric constant for any locally finite metric space and we study the property of having isoperimetric constant equal to zero. This property, called Small Neighborhood property, clearly extends amenability to any locally finite space. Therefore, we start making a comparison between this property and other notions of amenability for locally finite metric spaces that have been proposed by Gromov, Lafontaine and Pansu, by Ceccherini-Silberstein, Grigorchuk and de la Harpe and by Block and Weinberger. We discuss possible applications of the property SN in the study of embedding a metric space into another one. In particular, we propose three results: we prove that a certain class of metric graphs that are isometrically embeddable into Hilbert spaces must have the property SN. We also show, by a simple example, that this result is not true replacing property SN with amenability. As a second result, we prove that *many* spaces with *uniform bounded geometry* having a bi-lipschitz embedding into Euclidean spaces must have the property SN. Finally, we prove a Bourgain-like theorem for metric trees: a metric tree with uniform bounded geometry and without property SN does not have bi-lipschitz embeddings into finite-dimensional Hilbert spaces.

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