## Mathematics > Combinatorics

## Specified Intersections

Dhruv Mubayi, Vojtech Rodl

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Let $M$ be a subset of $\{0, \ldots, n\}$ and $F$ be a family of subsets of an $n$ element set such that the size of $A$ intersection $B$ is in $M$ for every $A$, $B$ in $F$. Suppose that I is the maximum number of consecutive integers contained in $M$ and $n$ is sufficiently large. Then we prove that $|F|<\min \left\{1.622^{\wedge} n 100^{\wedge}, 2^{\wedge}\left\{n / 2+l \log ^{\wedge} 2 n\right\}\right\}$.
The first bound complements the previous bound of roughly (1.99)^n due to Frankl and the second author which applies even when $M=\{0,1, . ., n\}-\{n / 4\}$. For small I, the second bound above becomes better than the first bound. In this case, it yields $2^{\wedge}\{n / 2+o(n)\}$ and this can be viewed as a generalization (in an asymptotic sense) of the famous Eventown theorem of Berlekamp.

Our second result complements the result of Frankl-Rodl in a different direction. Fix eps>0 and eps $n<t<n / 5$ and let $M=\{0,1, . ., n)-\left(t, t+n^{\wedge}\{0.525\}\right)$. Then, in the notation above, we prove that for $n$ sufficiently large, $|\mathrm{F}|<\mathrm{n}\{\mathrm{n}$ \choose ( $\mathrm{n}+\mathrm{t}$ )/2\}.
This is essentially sharp aside from the multiplicative factor of $n$. The short proof uses the Frankl-Wilson theorem and results about the distribution of prime numbers.

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