Mathematics > Combinatorics

The adjacency matroid of a graph

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If G is a looped graph, then its adjacency matrix represents a binary matroid $M_{A}(G)$ on $V(G)$. $M_{A}(G)$ may be obtained from the delta-matroid represented by the adjacency matrix of G, but $M_{A}(G)$ is less sensitive to the structure of G. Jaeger proved that every binary matroid is $M_{A}(G)$ for some G [Ann. Discrete Math. 17 (1983), 371-376]. The relationship between the matroidal structure of $M_{A}(G)$ and the graphical structure of G has many interesting features. For instance, the matroid minors $M_{A}(G)-v$ and $M_{A}(G)/v$ are both of the form $M_{A}(G^{\prime}-v)$ where $G^{\prime}$ may be obtained from G using local complementation. In addition, matroidal considerations lead to a principal vertex tripartition, distinct from the principal edge tripartition of Rosenstiehl and Read [Ann. Discrete Math. 3 (1978), 195-226]. Several of these results are given two very different proofs, the first involving linear algebra and the second involving set systems or delta-matroids. Also, the Tutte polynomials of the adjacency matroids of G and its full subgraphs are closely connected to the interlace polynomial of Arratia, Bollob\'{a}s and Sorkin [Combinatorica 24 (2004), 567-584].

Comments:	v1: 19 pages, 1 figure. v2: 20 pages, 1 figure. v3:29 pages, no figures. v3 includes an account of the relationship between the adjacency matroid of a graph and the delta-matroid of a graph. v4: 30 pages, 1 figure. v5: 31 pages, 1 figure. v6: 38 pages, 3 figures. v6 includes a discussion of the duality between graphic matroids and adjacency matroids of looped circle graphs
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