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Remarks for the Ramsey theory for trees

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Extending Furstenberg's ergodic theoretic proof for Szemer\'edi's theorem on arithmetic progressions, Furstenberg and Weiss (2003) proved the following qualitative result. For every d and k, there exists an integer N such that no matter how we color the vertices of a complete binary tree T_N of depth N with k colors, we can find a monochromatic replica of T_d in T_N such that (1) all vertices at the same level in T_d are mapped into vertices at the same level in T_N; (2) if a vertex x of T_d is mapped into a vertex y in T_N, then the two children of x are mapped into descendants of the the two children of y in T_N, respectively; and 3 the levels occupied by this replica form an arithmetic progression. This result and its density versions imply van der Waerden's and Szemer\'edi's theorems, and laid the foundations of a new Ramsey theory for trees.
Using simple counting arguments and a randomized coloring algorithm called random split, we prove the following related result. Let N=N(d,k) denote the smallest positive integer such that no matter how we color the vertices of a complete binary tree T_N of depth N with k colors, we can find a monochromatic replica of T_d in T_N which satisfies properties (1) and (2) above. Then we have N(d,k)=\Theta(dk\log k). We also prove a density version of this result, which, combined with Szemer\'edi's theorem, provides a very short combinatorial proof of a quantitative version of the Furstenberg-Weiss theorem.

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