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Bounds for graph regularity and removal lemmas

David Conlon, Jacob Fox

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We show, for any positive integer k, that there exists a graph in which any equitable partition of its vertices into k parts has at least ck^2/\log^* k pairs of parts which are not \epsilon-regular, where c,\epsilon>0 are absolute constants. This bound is tight up to the constant c and addresses a question of Gowers on the number of irregular pairs in Szemer\'edi's regularity lemma. In order to gain some control over irregular pairs, another regularity lemma, known as the strong regularity lemma, was developed by Alon, Fischer, Krivelevich, and Szegedy. For this lemma, we prove a lower bound of wowzer-type, which is one level higher in the Ackermann hierarchy than the tower function, on the number of parts in the strong regularity lemma, essentially matching the upper bound. On the other hand, for the induced graph removal lemma, the standard application of the strong regularity lemma, we find a different proof which yields a tower-type bound.

We also discuss bounds on several related regularity lemmas, including the weak regularity lemma of Frieze and Kannan and the recently established regular approximation theorem. In particular, we show that a weak partition with approximation parameter \epsilon may require as many as 2^{\Omega (\epsilon^{-2})} parts. This is tight up to the implied constant and solves a problem studied by Lov\'asz and Szegedy.

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