Chromatic number, clique subdivisions, and the conjectures of Hajós and Erdős-Fajtlowicz

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For a graph G, let $\chi(G)$ denote its chromatic number and $\sigma(G)$ denote the order of the largest clique subdivision in G. Let H(n) be the maximum of $\chi(G)/\sigma(G)$ over all n-vertex graphs G. A famous conjecture of Haj\'os from 1961 states that $\sigma(G) \geq \chi(G)$ for every graph G. That is, $H(n) \leq 1$ for all positive integers n. This conjecture was disproved by Catlin in 1979. Erd\H{o}s and Fajtlowicz further showed by considering a random graph that $H(n) \geq cn^{1/2}/\log n$ for some absolute constant $c>0$. In 1981 they conjectured that this bound is tight up to a constant factor in that there is some absolute constant C such that $\chi(G)/\sigma(G) \leq Cn^{1/2}/\log n$ for all n-vertex graphs G. In this paper we prove the Erd\H{o}s-Fajtlowicz conjecture. The main ingredient in our proof, which might be of independent interest, is an estimate on the order of the largest clique subdivision which one can find in every graph on n vertices with independence number α.

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