库仑费用多边际最优输运问题的全局优化算法（刘歆）

2023-12-13

In this work, we construct a novel numerical method for solving the multimarginal optimal transport problems with Coulomb cost. This type of optimal transport problem arises in quantum physics and plays an important role in understanding the strongly correlated quantum systems. With a Monge-like ansatz, we transfer the original high-dimensional problems into mathematical programmings with generalized complementarity constraints, and thus the curse of dimensionality is surmounted. However, the latter ones are themselves hard to deal with from both theoretical and practical perspectives. Moreover, in the presence of nonconvexity, brute-force searching for global solutions becomes prohibitive as the problem size grows large. To this end, we propose a global optimization approach for solving the nonconvex optimization problems, by exploiting an efficient proximal block coordinate descent local solver and an initialization subroutine based on hierarchical grid refinements. We conduct numerical simulations on some typical physical systems to show the efficiency of our approach. The results match well with both theoretical predictions and physical intuitions and provide indications for Monge solutions in two-dimensional contexts. In addition, we give the first visualization of approximate optimal transport maps for some two-dimensional systems.

Publication:

SIAM Journal on Scientific Computing, Volume 45, Issue 3 (2023)

http://dx.doi.org/10.1137/21M1455164

Author:

Yukuan Hu

State Key Laboratory of Scientific and Engineering Computing, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, and University of Chinese Academy of Sciences, Beijing, China.

Huajie Chen

School of Mathematical Sciences, Beijing Normal University, Beijing, China.

Xin Liu

State Key Laboratory of Scientific and Engineering Computing, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, and University of Chinese Academy of Sciences, Beijing, China.

Email: liuxin@lsec.cc.ac.cn

中国科学院 CHINESE ACADEMY OF SCIENCES

版权所有 © 中国科学院数学与系统科学研究院 备案号：京ICP备05002806-1号 京公网安备110402500020号
电话：86-10-82541777 传真：86-10-82541972 Email：contact@amss.ac.cn
地址：北京市海淀区中关村东路55号 邮政编码：100190