



向量优化及其若干进展

戎卫东¹, 杨新民^{2,*}

1. 内蒙古大学数学科学学院, 呼和浩特 010021; 2. 重庆师范大学数学科学学院, 重庆 400047

Vector optimization and its developments

RONG Weidong¹, YANG Xinmin^{2,*}

1. College of Mathematics Science, Inner Mongolia University, Hohhot 010021, China; 2. College of Mathematics Science, Chongqing Normal University, Chongqing 400047, China

- 摘要
- 参考文献
- 相关文章

Download: PDF (854KB) HTML (1KB) Export: BibTeX or EndNote (RIS) Supporting Info

摘要 在一定的约束条件下极小化或极大化向量值函数, 这就是向量优化。向量优化是数学规划学科中的重要分支学科, 是具有重要应用价值的、新兴的和多学科交叉的研究领域。自1950年以来, 已经逐步形成较完整的理论体系, 算法研究也有一定的进展, 应用日渐广泛。简述了它的发展历程、主要特征、基本理论和方法, 综述了国内学者近几年来在若干领域的发展状况和主要代表性成果, 展望了向量优化学科未来的发展方向。

关键词: 向量优化 学科概述 学科发展现状 研究展望

Abstract: Vector optimization is a mathematical model which minimizes or maximizes a vector-valued function. As an important part of mathematical programming, vector optimization is a promising interdisciplinary research field with many significant applications. Since 1950, the structure of the theory of vector optimization has been very complete, as well as some important progresses have been made in the study of methods, furthermore the applications have been flourishing. In this paper, we briefly review the developments of vector optimization, introduce the main characteristics, the basic theory and methods of it, highlight some recent typical progresses achieved by Chinese researchers, and propose some possible research prospects in future.

Keywords: vector optimization, overview of subjects, present situation of academic development, research prospects

基金资助:

国家自然科学基金 (No. 11271391), 运筹学与系统工程重庆市市级重点实验室专项课题 (No. 956806)

通讯作者 杨新民 Email: xmyang@cqnu.edu.cn

引用本文:

于永, 张欣, 刘桂真 .向量优化及其若干进展[J] 运筹学学报, 2014,V18(1): 9-38

YU Yong, ZHANG Xin, LIU Gui-Zhen .Vector optimization and its developments[J] OR TRANSACTIONS, 2014,V18(1): 9-38

链接本文:

http://202.120.127.195/shu_ycxxb/CN/ 或 http://202.120.127.195/shu_ycxxb/CN/Y2014/V18/I1/9

Service

- ▶ 把本文推荐给朋友
- ▶ 加入我的书架
- ▶ 加入引用管理器
- ▶ Email Alert
- ▶ RSS

作者相关文章

- ▶ 于永
- ▶ 张欣
- ▶ 刘桂真

[1] 陈光亚. 向量优化问题某些基础理论及其发展 [J]. 重庆师范大学学报 (自然科学版), 2005, 22(3): 1-4.

[2] Jahn J. Vector Optimization: Theory, Applications and Extensions [M]. New York: Springer-Verlag, 2011.

[3] Craven B D, Mond B. Transposition theorems for cone-convex functions [J]. SIAM Journal on Applied Mathematics, 1973, 24: 603-612.

[4] Hayashi M, Komiya H. Perfect duality for convexlike programs [J]. Journal of Optimization Theory and Applications, 1982, 38: 269-275.

[5] Jeyakumar V. A generalization of a minimax theorem of Fan via a theorem of the alternative [J]. Journal of Optimization Theory and Applications, 1986, 48: 525-533.

- [6] Yang X M. Alternative theorems and optimality conditions with weakened convexity [J]. Opsearch, 1992, 29(2): 125-135.
- [7] Song W. Lagrangian duality for minimization of nonconvex multifunctions [J]. Journal of Optimization Theory and Applications, 1997, 93: 167-182. 
- [8] Yang X M, Li D, Wang S Y. Near-subconvexlikeness in vector optimization with set-valued functions [J]. Journal of Optimization Theory and Applications, 2001, 110: 413-427. 
- [9] Sach P H. New generalized convexity notion for set-valued maps and application to vector optimization [J]. Journal of Optimization Theory and Applications, 2005, 125: 157-179. 
- [10] Alders C D, Sposito V A. On a sufficient optimality condition over convex feasible regions [J]. Bulletin of the Australian Mathematical Society, 1977, 16: 199-202. 
- [11] Luc D T. Theory of vector optimization [M]// Lecture Notes in Economic and Mathematical Systems, Berlin: Springer-Verlag, 1989, 319.
- [12] Kuhn H W, Tucker A W. Nonlinear programming [C]//Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability. Berkeley: University of California Press, 1951, 481-492.
- [13] Geoffrion A M. Proper efficiency and the theory of vector maximization [J]. Journal Mathematical Analysis and Applications, 1968, 22: 618-630. 
- [14] Borwein J M. Proper efficient points for maximizations with respect to cones [J]. SIAM Journal on Control and Optimization, 1977, 15: 57-63. 
- [15] Benson H P. An improved definition of proper efficiency for vector maximization with respect to cones [J]. Journal Mathematical Analysis and Applications, 1979, 71: 232-241. 
- [16] Henig M I. Proper efficiency with respect to cones [J]. Journal of Optimization Theory and Applications, 1982, 36: 387-407. 
- [17] Borwein J M, Zhuang D M. Super efficiency in convex vector optimization [J]. ZOR Methods and Models of Operations Research, 1991, 35: 175-184. 
- [18] Zheng X Y. Proper efficiency in locally convex topological vector spaces [J]. Journal of Optimization Theory and Applications, 1997, 94: 469-486. 
- [19] Lorian P. Necessary condition for $\backslash varepsilon$ -optimality [J]. Mathematical Programming Study, 1982, 19: 140-152 
- [20] Isac G. Ekeland's principle and Pareto $\backslash varepsilon$ -efficiency [M]//Multi-Objective Programming and Goal Programming, Lecture Notes in Economic and Mathematical Systems, New York: Springer-Verlag, 1996, 432.
- [21] Tammer C. A generalization of Ekeland's variational principle [J]. Optimization, 1992, 25: 129-141. 
- [22] Staib T. On two generalizations of Pareto minimality [J]. Journal of Optimization Theory and Applications, 1988, 59(2): 289-306.
- [23] Rong W D, Ma Y. Properly efficient solutions of vector optimization problems with set-valued maps [J]. OR Transactions, 2000, 4(4): 21-32.
- [24] Yang X M, Yang X Q, Chen G Y. Theorems of the alternative and optimization with set-valued maps [J]. Journal of Optimization Theory and Applications, 2000, 107: 627-640. 
- [25] Li Z F, Benson proper efficiency in the vector optimization of set-valued maps [J]. Journal of Optimization Theory and Applications, 1998, 98 (3): 623-649.
- [26] Chen G Y, Rong W D. Characterizations of the Benson proper efficiency for nonconvex vector optimization [J]. Journal of Optimization Theory and Applications, 1998, 98(2): 365-384. 
- [27] Mangasarian O L. Second and higher order duality in nonlinear programming [J]. Journal Mathematical Analysis and Applications, 1975, 51: 607-620. 
- [28] Kim D S, Yun Y B. Second-order symmetric and self duality in multiobjective programming [J]. Applied Mathematics Letters, 1997, 10(2): 17-22.
- [29] Yang X M, Hou S H. Second-order symmetric duality in multiobjective programming [J]. Applied Mathematics Letters, 2001, 14: 587-592. 
- [30] Giannessi F. Theory of alternative, quadratic programs and complementarity problems [M]// Variational Inequalities and Complementarity Problems, New York: Wiley, 1980, 151-186.
- [31] Chen G Y, Cheng G M. Vector variational inequality and vector optimization [M]//Toward Interactive and Intelligent Decision Support Systems, Lecture Notes in Economics and Mathematical Systems, Heidelberg: Springer-Verlag, 1987, 408-416.
- [32] 陈光亚. 向量变分不等式及其在多目标最优化中的应用 [J]. 科学通报, 1987, 8: 561-563.
- [33] Chen G Y, Yang X Q. Vector complementarity problem and its equivalences with weak minimal element in ordered spaces [J]. Journal Mathematical Analysis and Applications, 1990, 153: 136-158. 
- [34] Chen G Y, Craven B D. A vector variational inequality and optimization over an efficient set [J]. ZOR Methods and Models of Operations Research, 1990, 3: 1-12.
- [35] Chen G Y. Existence of solution for a vector variational inequality: an extension of the Hartmann-Stampacchia theorem [J]. Journal of

- [36] Yang X M, Yang X Q, Teo K L. Some remarks on the Minty vector variational inequality [J]. Journal of Optimization Theory and Applications, 2004, 121: 193-201. 
- [37] Zheng X Y. Generalizations of a theorem of Arrow, Barankin, and Blackwell in topological vector spaces [J]. Journal of Optimization Theory and Applications, 1998, 96(1): 221-233. 
- [38] 傅万涛. 集值映射多目标规划问题的解集的连通性 [J]. 高校应用数学学报, 1994, 9(3): 321-328.
- [39] Gutierrez C, Jimenez B, Novo V. A unified approach and optimality conditions for approximate solutions of vector optimization problems [J]. SIAM Journal on Optimization, 2006, 17: 688-710. 
- [40] Gao Y, Yang X M, Teo K L. Optimality conditions for approximate solutions of vector optimization problems [J]. Journal of Industrial and Management Optimization, 2011, 7: 483-496. 
- [41] Zhao K Q, Yang X M. E-Benson proper efficiency in vector optimization [J]. Optimization, Doi: org/10.1080/02331934.2013.798321.
- [42] Zhao K Q, Yang X M. E-proper saddle points and E-proper Duality in Vector Optimization with set-valued maps [J]. Taiwanese Journal of Mathematics, Doi: 10.11650/tjm.17.2013.3473.
- [43] Zheng X Y, Ng K F. A unified separation theorem for closed sets in a Banach space and optimality conditions for vector optimization [J]. SIAM Journal on Optimization, 2011, 21: 886-911. 
- [44] Qiu Q S, Yang X M. Some properties of approximate solutions for vector optimization problem with set-valued functions [J]. Journal of Global Optimization, 2010, 47: 1-12. 
- [45] Qiu Q S. Scalarization of approximate solution for vector equilibrium problems [J]. Journal of Industrial and Management Optimization, 2013, 9 (1): 143-151.
- [46] Gao Y, Hou S H, Yang X M. Existence and optimality conditions for approximate solutions to vector optimization problems [J]. J Opt Theory Appl, 2012, 152: 97-120. 
- [47] Liu C P, Lee H W. Lagrange multiplier rules for approximate solutions in vector optimization [J]. Journal of Industrial and Management Optimization, 2012, 8(3): 749-764. 
- [48] Chen C R, Li S J, Fang Z M. On the solution semicontinuity to a parametric generalized vector quasivariational inequality [J]. Computers and Mathematics with Applications, 2010, 60: 2417-2425. 
- [49] Li X B, Li S J. Continuity of approximate solution mappings for parametric equilibrium problems [J]. Journal of Global Optimization, 2011, 51: 541-548. 
- [50] Li S J, Liu H M, Zhang Y, et al. Continuity of the solution mappings to parametric generalized strong vector equilibrium problems [J]. Journal of Global Optimization, 2013, 55: 597-610. 
- [51] Xu Y D, Li S J. On the lower semicontinuity of the solution mappings to a parametric generalized strong vector equilibrium problem [J]. Positivity, 2013, 17: 341-353. 
- [52] Li S J, Li X B, Wang L N, et al. The HV^o-lder continuity of solutions to generalized vector equilibrium problems [J]. European Journal of Operational Research, 2009, 199: 334-338. 
- [53] Fang Y P, Huang N J, Yao J C. Well-posedness of mixed variational inequalities, inclusion problems and fixed point problems [J]. Journal of Global Optimization, 2008, 41: 117-133. 
- [54] Fang Y P, Huang N J, Yao J C. Well-posedness by perturbations of mixed variational inequalities in Banach spaces [J]. European Journal of Operational Research, 2010, 201(3): 682-692. 
- [55] Huang X X, Yang X Q. Generalized Levitin-Polyak well-posedness in constrained optimization [J]. SIAM Journal on Optimization, 2006, 17: 243-258. 
- [56] Huang X X, Yang X Q. Further study on the Levitin-Polyak well-posedness of constrained convex vector optimization problems [J]. Nonlinear Analysis: Theory, Methods and Applications, 2012, 75: 1341-1347. 
- [57] Huang X X, Yang X Q. Levitin-Polyak well-posedness for vector variational inequalities with functional constraints [J]. Numerical Functional Analysis and Optimization, 2010, 31: 440-459. 
- [58] Luo H L, Huang X X, Peng J W. Generalized well-posedness in convex vector optimization [J]. Pacific Journal of Optimization, 2011, 7: 353-367.
- [59] Peng J W, Wang Y, Wu S Y. Levitin-Polyak well-posedness for vector quasi-equilibrium problems with functional constraints [J]. Taiwanese Journal of Mathematics, 2012, 16(2): 635-649.
- [60] Peng J W, Wu S Y, Wang Y. Levitin-Polyak well-posedness of generalized vector quasi-equilibrium problems with functional constraints [J]. Journal of Global Optimization, 2012, 52(4): 779-795. 
- [61] Gong X H. Efficiency and Henig efficiency for vector equilibrium problems [J]. Journal of Optimization Theory and Applications, 2001, 108: 139-154. 

- [62] Gong X H. Connectedness of the solution sets and scalarization for vector equilibrium problems [J]. *Journal of Optimization Theory and Applications*, 2007, 133: 151-161. 
- [63] Qiu Q S, Yang X M. Connectedness of Henig weakly efficient solution set for set-valued optimization problems [J]. *Journal of Optimization Theory and Applications*, 2012, 152: 439-449. 
- [64] Chen C, Cheng T C E, Li S J, et al. Nonlinear augmented Lagrangian for nonconvex multiobjective optimization [J]. *Journal of Industrial and Management Optimization*, 2011, 7(1): 157-174. 
- [65] Zhou Y Y, Yang X Q. Augmented Lagrangian functions for constrained optimization problems [J]. *Journal of Global Optimization*, 2012, 52(1): 95-108. 
- [66] Li S J, Sun X K, Zhu S K. Higher-order optimality conditions for strict minimality in set-valued optimization [J]. *Journal of Nonlinear and Convex Analysis*, 2012, 13: 281-291.
- [67] Zhou Z A, Yang X M. Optimality conditions of generalized subconvexlike set-valued optimization problems based on the quasi-relative interior [J]. *Journal of Optimization Theory and Applications*, 2011, 150: 327-340. 
- [68] Li Z M. The optimality conditions for vector optimization of set-valued maps [J]. *Journal of Mathematical Analysis and Application*, 1999, 237: 413-424. 
- [69] Zhou Z A, Yang X M, Peng J W. Optimality conditions of set-valued optimization problems involving relative algebraic interior in ordered linear spaces [J]. *Optimization*, Doi: 10.1080/0233-1934-2012.656117.
- [70] Zhou Z A, Yang X M, Peng J W. Optimality conditions of vector optimization problems with set-valued maps based on the algebraic interior in real linear spaces [J]. *Optimization Letters*, Doi: 10.1007/s11590-013-0620-y.
- [71] Li T Y, Xu Y H. The strictly efficient subgradient of set-valued optimization [J]. *Bulletin of the Australian Mathematical Society*, 2007, 75: 361-371. 
- [72] Zhou Z A, Yang X M, Peng J W. \backslash arepsilon-strict subdifferentials of set-valued maps and optimality conditions [J]. *Nonlinear Analysis*, 2012, 75: 3761-3775. 
- [73] Zheng X Y, Yang X Q. Conic positive definiteness and sharp minima of fractional orders in vector optimization problems [J]. *Journal of Mathematical Analysis and Applications*, 2012, 391(2): 619-629. 
- [74] Zhang Y, Li S J. Minimax theorems for scalar set-valued mappings with nonconvex domains and applications [J]. *Journal of Global Optimization*, 2013, 57: 1359-1373. 
- [75] Zhang Y, Li S J. Minimax problems of uniformly same-order set-valued mappings [J]. *Bulletin of the Korean Mathematical Society*, 2013, 5: 1639-1650.
- [76] Zheng X Y, Ng K F. Metric subregularity and calmness for nonconvex generalized equations in Banach spaces [J]. *SIAM Journal on Optimization*, 2010, 20: 2119-2136. 
- [77] Yang X M, Yang X Q, Teo K L. Non-differentiable second order symmetric duality in mathematical programming with F-convexity [J]. *European Journal of Operational Research*, 2003, 144(3): 554-559. 
- [78] Yang X M, Yang X Q, Teo K L. Higher-order generalized convexity and duality in nondifferentiable multiobjective mathematical programming [J]. *Journal of Mathematical Analysis and Applications*, 2004, 297(1): 48-55. 
- [79] Yang X M, Yang X Q, Teo K L. Mixed type converse duality in multiobjective programming problems [J]. *Journal Mathematical Analysis and Applications*, 2005, 304(2): 394-398. 
- [80] Yang X M, Yang X Q, Teo K L, et al. Second order symmetric duality in non-differentiable multiobjective programming with F-convexity [J]. *European Journal of Operational Research*, 2005, 164(2): 406-416. 
- [81] Yang X M, Yang X Q, Teo K L, et al. Multiobjective second-order symmetric duality with F-convexity [J]. *European Journal of Operational Research*, 2005, 165(3): 585-591. 
- [82] Yang X M, Yang X Q, Teo K L. Converse duality in nonlinear programming with cone constraints [J]. *European Journal of Operational Research*, 2006, 170(2): 350-354. 
- [83] Zheng X Y, Yang X Q. The structure of weak sharp Pareto solution sets in piecewise linear multiobjective optimization in normed spaces [J]. *Sciences in China*, 2008, 51(7): 1243-1256.
- [84] Zheng X Y. Pareto solutions of polyhedral-valued vector optimization problems in Banach spaces [J]. *Set-Valued Analysis*, 2009, 17: 389-408. 
- [85] Fang Y P, Meng K W, Yang X Q. Piecewise linear multicriteria programs: the continuous case and its discontinuous generalization [J]. *Operations Research*, 2012, 60(2), 398-409.
- [86] Fang Y P, Huang N J, Yang X Q. Local smooth representations of parametric semiclosed polyhedra with applications to sensitivity in piecewise linear programs [J]. *Journal of Optimization Theory and Applications*, 2012, 155(3), 810-839.
- [87] Gong X H. Continuity of the solution set to parametric weak vector equilibrium problems [J]. *Journal of Optimization Theory and Applications*, 2008, 139: 35-46. 

- [88] Gong X H. Lower semicontinuity of the set of efficient solutions for generalized systems [J]. Journal of Optimization Theory and Applications, 2007, 138: 197-205.
- [89] Peng Z Y, Yang X M. Semicontinuity of the solution mappings to weak generalized parametric Ky Fan inequality problems with trifunctions [J]. Optimization, Doi: 10.1080/ 02331934.2012.~660693.
- [90] Li S J, Meng K W. Calculus rules for derivatives of multimaps [J]. Set-Valued and Variational Analysis, 2009, 17: 21-39. 
- [91] Li S J, Li M H. Sensitivity analysis of parametric weak vector equilibrium problems [J]. Journal of Mathematical Analysis and Applications, 2011, 380: 354-362. 
- [92] Li M H, Li S J. Robinson metric regularity of parametric variational systems [J]. Nonlinear Analysis: Theory, Methods and Applications, 2011, 74: 2262-2271. 
- [93] Xue X W, Li S J. Coderivatives of the generalized perturbation maps [J]. Positivity, 2011, 15: 309-329. 
- [94] Li S J, Penot J P, Xue X W. Codifferential calculus [J]. Set-Valued and Variational Analysis, 2011, 19: 505-536. 
- [95] Li J, Huang N J, Yang X Q. Weak sharp minima for set-valued vector variational inequalities with an application [J]. European Journal of Operational Research , 2010, 205: 262-272. 
- [96] Huang N J, Li J, Yang X Q. Weak sharpness for gap functions in vector variational inequalities [J]. Journal of Mathematical Analysis and Applications, 2012, 394(2): 449-457. 
- [97] Zhou L W, Huang N J. Generalized KKM theorems on Hadamard manifolds with applications [EB/OL]. [2013-10-20]. <http://www.paper.edu.cn/index.php/default/releasepaper/content/200906-669>. 浏览
- [98] Zhou L W, Huang N J. Existence of solutions for vector optimization on Hadamard manifolds [J]. Journal of Optimization Theory and Applications, 2013, 157: 44-53. 
- [99] Li X B, Huang N J. Generalized vector quasi-equilibrium problems on Hadamard manifolds [J]. Optimization Letters, Doi: 10.1007/s11590-013-0703-9.
- [100] Chen Z, Huang X X, Yang X Q. Generalized proximal point algorithms for multiobjective optimization problems [J]. Journal of Applied Mathematics, 2011, 90(6): 935-949.
- [101] Chen Z, Huang H Q, Zhao K Q. Approximate generalized proximal-type method for convex vector optimization problem in Banach spaces [J]. Computers and Mathematics with Applications, 2009, 57(7): 1196-1203. 
- [102] Chen Z, Xiang C H, Zhao K Q, et al. Convergence analysis of Tykhonov-type regularization algorithms for multiobjective optimization problems [J]. Applied Mathematics and Computation, 2009, 211(1): 167-172. 
- [103] 徐家旺, 黄小原. 鲁棒优化研究的新进展 [C]//中国企业运筹学学术交流大会论文集, 2007, 20-26.
- [104] 李亚林, 陈静, 罗彪, 等. 一种求解鲁棒优化问题的多目标进化方法 [J]. 计算机工程与应用, 2011, 47(24): 58-62.
- [105] 邓乃阳, 田英杰. 数据挖掘中的新方法-----支持向量机 [M]. 北京: 科学出版社, 2004.
- [106] 袁玉波, 杨传胜, 黄廷祝. 数据挖掘与最优化技术及其应用 [M]. 北京: 科学出版社, 2007.
- [107] Yu P L. Cone convexity, cone extreme points, and nondominated solutions in decision problems with multiobjectives [J]. Journal of Optimization Theory and Applications, 1974, 14: 319-377. 
- [108] Chen G Y, Huang X X. A unified approach to the existing three types of variational principles for vector valued functions [J]. Mathematical Methods of Operations Research, 1998, 48: 349-357. 
- [1] 中国运筹学会数学规划分会.中国数学规划学科发展概述[J].运筹学学报, 2014,18(1): 1-8
- [2] 龚舒, 龚循华.向量均衡问题的超有效性的对偶形式[J].运筹学学报, 2013,17(2): 107-123

