变分不等式与互补问题、双层规划与平衡约束数学规划问题的若干进展

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Several developments of variational inequalities and complementarity problems, bilevel programming and MPEC

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摘要 考虑有限维变分不等式与互补问题、双层规划以及均衡约束的数学规划问题. 在简单介绍这些问题之后，重点介绍近年来这些领域中发展迅速的几个研究方向，包括对称锥互补问题的理论与算法、变分不等式的投影收缩算法、随机变分不等式与随机互补问题的模型与方法、双层规划以及均衡约束数学规划问题的新方法. 最后提出几个进一步研究的方向.

关键词： 变分不等式   互补问题   双层规划   均衡约束的数学规划问题

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Abstract： This paper investigates finite-dimensional variational inequalities and complementarity problems, bilevel programming problems and mathematical programs with equilibrium constraints (MPECs). After a brief introduction to these problems, this paper focuses on several recent rapidly developing aspects in these fields, which include theories and methods for symmetric cone complementarity problems, projection and contraction methods for variational inequality problems, models and methods for stochastic variational inequalities and stochastic complementarity problems, and new methods for bilevel programming problems and MPECs. Finally, several future research directions are proposed in this paper.

Keywords： variational inequality,   complementarity problem,   bilevel programming,   mathematical program with equilibrium constraints

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