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随机市场中均值-方差模型最优投资策略的时间不相容性及其修正

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Time inconsistency of the mean-variance optimal policy in stochastic markets and its revision

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摘要 由于方差算子在动态规划意义下不可分，导致随机市场中多期均值-方差模型的最优投资策略不满足时间相容性，即Bellman最优性原理. 为此，首先提出了随机市场中比Bellman最优性原理更弱的时间相容性，并证明在投资区间的任意中间时刻，当投资者的财富不超过某一给定的财富阈值时，最优投资策略满足弱时间相容性；当投资者的财富超过该阈值时，最优投资策略将不再是弱时间相容的，且导致投资者变为非理性，即他会同时极小化终期财富的均值和方差. 在这种情形下，通过放松自融资约束，对最优投资策略进行了修正，使得其满足：修正策略可使投资者回归理性；相对于终期财富，修正策略可以获得与最优投资策略相同的均值和方差. 在策略修正过程中，投资者可以从市场中获得一个严格正的现金流. 这些结果表明修正策略要优于原最优投资策略，拓展了现有关于确定市场下多期均值-方差模型的求解以及策略时间相容性的结论.

关键词： 时间不相容性 均值-方差 随机市场 Bellman最优性原理 策略修正

Abstract： Due to the non-separability of the variance operator in sense of dynamic programming, the optimal investment policy of the multi-period mean-variance model in stochastic markets doesn't satisfy the time consistency corresponding to Bellman's optimality principle. To overcome this shortcoming, we first propose a new time consistency in stochastic markets, which is weaker than Bellman's optimality principle, and prove that the optimal investment policy satisfies the weak time consistency at any intermediate period as long as the investor's wealth is no more than a specific threshold. When the investor's wealth exceeds the given threshold, the weak time consistency no longer holds, and the investor becomes irrational since he minimizes mean and variance of the terminal wealth at the same time. To avoid this, by relaxing the self-financing constraint, we revise the optimal investment policy so that the revised policy can make the investor rational and can attain the same mean and variance of the terminal wealth as those of the optimal investment policy. It is shown that, in this process, a positive cash flow should be taken out of the market. These conclusions show that the revised policy is more efficient than the optimal investment policy. Our results extend existing conclusions for the multi-period mean-variance model in deterministic markets and the understanding about time consistency.

Keywords： time inconsistency, mean-variance, stochastic markets, Bellman's optimality principle, policy revision

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