

# Single-crossover recombination and ancestral recombination trees

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We consider the Wright-Fisher model for a population of  $N$  individuals, each identified with a sequence of a finite number of sites, and single-crossover recombination between them. We trace back the ancestry of single individuals from the present population. In the  $N \rightarrow \infty$  limit without rescaling of parameters or time, this ancestral process is described by a random tree, whose branching events correspond to the splitting of the sequence due to recombination. With the help of a decomposition of the trees into subtrees and an inclusion-exclusion principle, we find a closed-form expression for the probabilities of the topologies of the ancestral trees. At the same time, these probabilities lead to an explicit solution of the deterministic single-crossover equation. The latter is a discrete-time dynamical system that emerges from the Wright-Fisher model via a law of large numbers and has been waiting for a solution for many decades.

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