From almost sure local regularity to almost sure Hausdorff dimension for Gaussian fields

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Fine regularity of stochastic processes is usually measured in a local way by local H\"older exponents and in a global way by fractal dimensions. Following a previous work of Adler, we connect these two concepts for multiparameter Gaussian random fields. More precisely, we prove that almost surely the Hausdorff dimensions of the range and the graph in any ball $B(t_0,\rho)$ are bounded from above using the local H\"older exponent at t_0. We define the deterministic local sub-exponent of Gaussian processes, which allows to obtain an almost sure lower bound for these dimensions. Moreover, the Hausdorff dimensions of the sample path on an open interval are controlled almost surely by the minimum of the local exponents.
Then, we apply these generic results to the cases of the multiparameter fractional Brownian motion, the multifractional Brownian motion whose regularity function H is irregular and the generalized Weierstrass function, whose Hausdorff dimensions were unknown so far.

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