Mathematics > Probability

# Random determinants, mixed volumes of ellipsoids, and zeros of Gaussian random fields

Zakhar Kabluchko, Dmitry Zaporozhets

*(Submitted on 2 Jun 2012)*

Consider a $d\times d$ matrix $M$ whose rows are independent centered non-degenerate Gaussian vectors $\xi_1,...,\xi_d$ with covariance matrices $\Sigma_1,...,\Sigma_d$. Denote by $\mathcal{E}_i$ the location-dispersion ellipsoid of $\xi_i:\mathcal{E}_i={\mathbf{x}\in\mathbb{R}^d : \mathbf{x}^\top\Sigma_i^{-1} \mathbf{x}\leqslant1}$. We show that $$ \mathbb{E}\,|\det M|=\frac{d!}{(2\pi)^{d/2}}V_d(\mathcal{E}_1,...,\mathcal{E}_d), $$ where $V_d(\cdot,...,\cdot)$ denotes the {\it mixed volume}. We also generalize this result to the case of rectangular matrices. As a direct corollary we get an analytic expression for the mixed volume of $d$ arbitrary ellipsoids in $\mathbb{R}^d$. As another application, we consider a smooth centered non-degenerate Gaussian random field $X=(X_1,...,X_k)^\top:\mathbb{R}^d\to\mathbb{R}^k$. Using Kac-Rice formula, we obtain the geometric interpretation of the intensity of zeros of $X$ in terms of the mixed volume of location-dispersion ellipsoids of the gradients of $X_i/\sqrt{\mathbf{Var} X_i}$. This relates zero sets of equations to mixed volumes in a way which is reminiscent of the well-known Bernstein theorem about the number of solutions of the typical system of algebraic equations.

| | |
|---|---|
| Subjects: | **Probability (math.PR)**; Differential Geometry (math.DG) |
| MSC classes: | Primary 60B20, 52A39, Secondary 60G15, 53C65 |
| Cite as: | **arXiv:1206.0371 [math.PR]** |
| | (or **arXiv:1206.0371v1 [math.PR]** for this version) |

## Submission history

From: Dmitry Zaporozhets [view email]
**[v1]** Sat, 2 Jun 2012 12:24:01 GMT (8kb)

*Which authors of this paper are endorsers?*