

A CLT for Information-theoretic statistics of Non-centered Gram random matrices

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In this article, we study the fluctuations of the random variable:
$$\mathcal{I}_n(\rho) = \frac{1}{N} \log \det(\Sigma_n \Sigma_n^* + \rho I_N), \quad (\rho > 0)$$
 where $\Sigma_n = n^{-1/2} D_n^{1/2} X_n \tilde{D}_n^{1/2} + A_n$, as the dimensions of the matrices go to infinity at the same pace. Matrices X_n and A_n are respectively random and deterministic $N \times n$ matrices; matrices D_n and \tilde{D}_n are deterministic and diagonal, with respective dimensions $N \times N$ and $n \times n$; matrix $X_n = (X_{ij})$ has centered, independent and identically distributed entries with unit variance, either real or complex.

We prove that when centered and properly rescaled, the random variable $\mathcal{I}_n(\rho)$ satisfies a Central Limit Theorem and has a Gaussian limit. The variance of $\mathcal{I}_n(\rho)$ depends on the moment $E|X_{ij}|^2$ of the variables X_{ij} and also on its fourth cumulant $\kappa = E|X_{ij}|^4 - 2|E|X_{ij}|^2|^2$.

The main motivation comes from the field of wireless communications, where $\mathcal{I}_n(\rho)$ represents the mutual information of a multiple antenna radio channel. This article closely follows the companion article "A CLT for Information-theoretic statistics of Gram random matrices with a given variance profile", [arXiv:1107.0145v1](#).

Ann. Appl. Probab. (2008) by Hachem et al., however the study of the fluctuations associated to non-centered large random matrices raises specific issues, which are addressed here.

Subjects: **Probability (math.PR)**; Statistics Theory (math.ST)

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