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跳扩散过程下期权定价的数值方法

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Numerical method for option pricing under jump-diffusion process

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摘要 研究了跳扩散过程下期权价值所满足\,PIDE\,方程的数值计算方法. 利用四阶差分格式对空间离散, 引入四阶\,Lagrange\,插值多项式对边界进行延拓, 得到一个非齐次线性系统. 基于矩阵指数的\,$\mathrm{Pad\acute{e}}$\,逼近方法及其分数表示形式, 构建了一种高阶光滑\,Crank-Nicolson\,差分格式. 数值计算验证了该种方法的有效性, 讨论了跳跃强度对标准期权和障碍期权的影响. 与传统的\,Crank-Nicolson\,格式相比, 该格式很好地处理了在执行价格和障碍点附近数值震荡的问题. 该种方法亦可应用于一般具有非光滑边界的线性系统问题.

关键词: 期权 跳扩散过程 数值方法 $\mathrm{Pad\acute{e}}$\ 逼近 光滑\ Crank-Nicolson\ 格式

Abstract: Numerical method for partial integro-differential equation (PIDE) resulting from option value under jump-diffusion process was studied. A non-homogeneous linear system was obtained by discretizing the spatial derivatives utilizing the fourth-order difference and extending boundary using fourth-order Lagrange interpolating polynomial. Based on $\mathrm{Pad\acute{e}}$ approximations and partial fraction version of the matrix exponential, a high-order smoothing Crank-Nicolson scheme was constructed. Numerical calculation discussed the influence of jump intensity on vanilla option value and barrier option value, showed that the algorithm was efficient. Compared with classic Crank-Nicolson scheme, the numerical scheme avoided the spurious oscillation near the strike price and barrier value. The algorithm also can be used in the general linear boundary value problem which has non-smooth boundary.

Key words: option jump-diffusion process numerical method $\mathrm{Pad\acute{e}}$ approximation smoothing Crank-Nicolson scheme

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