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Parareal 算法的均方稳定性分析

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ANALYSIS OF MEAN-SQUARE STABILITY OF THE PARAREAL ALGORITHM

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摘要

Parareal 算法是一种非常有效的实时并行计算方法. 与传统的并行计算方法相比,该算法的显著特点是它的时间并行性 | 先将整个计算时间划分成若干个子区间,然后在每个子区间内同时进行计算. Parareal算法收敛速度快, 并行效率高, 且易于编程实现, 从 2001 年由 Lions,Maday 和 Turinici等人首次提出至今, 在短短的几年间得到了广泛的研究和应用. 最近, Parareal 算法在随机微分方程数值解中的应用也得到了一些学者的关注. 本文中, 我们研究 Parareal算法在随机微分方程数值解中的均方稳定性, 分析保持算法稳定的充分性条件. 通过分析, 我们得到了如下结论: a)Parareal 算法在有限时间区间内是超线性收敛的; b)在无限时间区间内, 该算法是线性收敛的. 最后, 通过数值试验, 我们验证了本文中的理论结果.

关键词: Parareal 算法 并行计算 稳定性 超线性收敛 线性收敛

Abstract:

Parareal algorithm is a very efficient parallel in time computation methods. Compared with traditional parallel methods, this algorithm has the advantages of faster convergence, higher parallel performance and easy coding. This algorithm was first proposed by Lions, Maday and Turinici in 2001 and has attracted many researchers over the past few years. Recently, the application and theoretical analysis of this algorithm for stochastic computation have been investigated by some researchers. In this paper, we analyze the Mean-square stability of the Parareal algorithm in stochastic computation. The sufficient conditions under which the Parareal algorithm is stable are obtained and it is shown that: a) the algorithm converges superlinearly on any bounded time interval and b) the convergence speed is only linear on unbounded time intervals. Finally, numerical results are given to validate our theoretical conclusions.

Key words: Parareal algorithm parallel computation stability superlinear convergence linear convergence

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