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Hardy Spaces $H_L^p({\mathbb R}^n)$ Associated to Operators Satisfying k-Davies-Gaffney Estimates

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Let L be a one to one operator of type ω having a bounded H_∞ functional calculus and satisfying the k-Davies-Gaffney estimates with $k\in{\mathbb N}$. In this paper, the authors introduce the Hardy space $H_L^p(\mathbb{R}^n)$ with $p\in (0,\,1]$ associated to L in terms of square functions defined via $\{e^{-t^{2k}L}\}_{t>0}$ and establish their molecular and generalized square function characterizations. Typical examples of such operators include the $2k$-order divergence form homogeneous elliptic operator L_1 with complex bounded measurable coefficients and the $2k$-order Schr\"odinger type operator $L_2\equiv (-\Delta)^k+V^k$, where Δ is the Laplacian and $0\le V\in L^k_{\mathop\mathrm{loc}}(\mathbb{R}^n)$. Moreover, as applications, for $i\in\{1,\,2\}$, the authors prove that the associated Riesz transform $\nabla^k(L_i^{-1/2})$ is bounded from $H_{L_i}^p(\mathbb{R}^n)$ to $H^p(\mathbb{R}^n)$ for $p\in(n/(n+k),\,1]$ and establish the Riesz transform characterizations of $H_{L_1}^p(\mathbb{R}^n)$ for $ p\in(rn/(n+kr),\,1]$ if $\{e^{-tL_1}\}_{t>0}$ satisfies the L^r-L^2 k-off-diagonal estimates with $r\in (1,2]$. These results when $k\equiv1$ and $L\equiv L_1$ are known.

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