

Cornell University Library We gratefully acknowledge support from the Simons Foundation and member institutions

arXiv.org > math > arXiv:1207.0086

Search or Article-id

All papers 🚽 Go!

(Help | Advanced search)

## Download:

- PDF
- PostScript
- Other formats

Current browse context: math.FA

< prev | next >

new | recent | 1207

Change to browse by:

math



Mathematics > Functional Analysis

# Semispectral Measures and Feller markov Kernels

### **Roberto Beneduci**

(Submitted on 30 Jun 2012)

It is well known \cite{B1,P} that a real positive semispectral measure \$F\$ is commutative if and only if there exist a self-adjoint operator \$A\$ and a Markov kernel \$\mu {(\cdot)}(\cdot):\sigma(A)\times\mathcal{B}(\mathbb{R})\to[0,1]\$ such that \$F(\Delta)=\mu\_{\Delta}(A)\$. In quantum mechanics, it is usual to meet commutative semispectral measures for which the functions \$\mu\_ {\Delta}:\sigma(A)\to[0,1]\$, \$\Delta\in\mathcal{B}(\mathbb{R})\$, are continuous (in which case  $\sum_{(\cdot)}(\cdot)$  is a strong Feller Markov kernel). An important example is the semispectral measure used in quantum mechanics to represent the unsharp position observable. In the present work we give a stronger characterization of commutative semispectral measures and study general conditions for the continuity of  $\sum_{\lambda \in A} (A)$ [0,1]\$. In particular, \bullet we show that \$F\$ is commutative if and only if there exist a self-adjoint operator \$A\$ and a Markov kernel \$\mu\_{(\cdot)} (\cdot):\Gamma\times\mathcal{B}(\mathbb{R})\to[0,1]\$, \$\Gamma\subset\sigma(A)\$, \$E(\Gamma)=\mathbf{1}\$, such that \$\$F(\Delta) =\int\_{\Gamma}\mu\_{\Delta}(\lambda)\,dE\_{\lambda},\$\$ \noindent and \$\mu\_ {(\Delta)}\$ is continuous for each \$\Delta\in R\$ where, \$R\subset\mathcal{B} (\mathbb{R})\$ is a ring which generates the Borel \$\sigma\$-algebra of the reals \$\mathcal{B}(\mathbb{R})\$. Moreover, \$\mu\_{(\cdot)}(\cdot)\$ is a Feller Markov kernel and separates the points of \$\Gamma\$. \bullet we prove that \$F\$ admits a strong Feller Markov kernel \$\mu\_{(\cdot)}(\cdot)\$, if and only if \$F\$ is uniformly continuous. Finally, we prove that if \$F\$ is absolutely continuous with respect to a regular finite measure \$\nu\$ then, it admits a

strong Feller Markov kernel.

Subjects: Functional Analysis (math.FA)

MSC classes: 46L10, 81Q10, 46G10, 47L30, 46N50 Cite as: arXiv:1207.0086 [math.FA] (or arXiv:1207.0086v1 [math.FA] for this version)

### **Submission history**

From: Roberto Beneduci [view email] [v1] Sat, 30 Jun 2012 12:41:11 GMT (23kb)

#### Which authors of this paper are endorsers?

Link back to: arXiv, form interface, contact.