Mathematics > Functional Analysis

Semispectral Measures and Feller markov Kernels

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It is well known \cite{B1,P} that a real positive semispectral measure F is commutative if and only if there exist a self-adjoint operator A and a Markov kernel $\mu_{(\cdot)}(\cdot):\sigma(A)\times\mathcal{B}(\mathbb{R})\to[0,1]$ such that $F(\Delta)=\mu_{\Delta}(A)$. In quantum mechanics, it is usual to meet commutative semispectral measures for which the functions $\mu_{\Delta}:\sigma(A)\to[0,1]$, $\Delta\in\mathcal{B}(\mathbb{R})$, are continuous (in which case $\mu_{(\cdot)}(\cdot)$ is a strong Feller Markov kernel). An important example is the semispectral measure used in quantum mechanics to represent the unsharp position observable. In the present work we give a stronger characterization of commutative semispectral measures and study general conditions for the continuity of $\mu_{\Delta}:\sigma(A)\to[0,1]$. In particular, \bullet we show that F is commutative if and only if there exist a self-adjoint operator A and a Markov kernel $\mu_{(\cdot)}(\cdot):\Gamma\times\mathcal{B}(\mathbb{R})\to[0,1]$, $\Gamma\subset\sigma(A)$, $E(\Gamma)=\mathbf{1}$, such that $$F(\Delta)=\int_{\Gamma}\mu_{\Delta}(\lambda)\,dE_{\lambda},$$ \noindent and $\mu_{(\Delta)}$ is continuous for each $\Delta\in R$ where, $R\subset\mathcal{B}(\mathbb{R})$ is a ring which generates the Borel σ-algebra of the reals $\mathcal{B}(\mathbb{R})$. Moreover, $\mu_{(\cdot)}(\cdot)$ is a Feller Markov kernel and separates the points of Γ. \bullet we prove that F admits a strong Feller Markov kernel $\mu_{(\cdot)}(\cdot)$, if and only if F is uniformly continuous. Finally, we prove that if F is absolutely continuous with respect to a regular finite measure ν then, it admits a strong Feller Markov kernel.

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