Mathematics > Functional Analysis

Formulating Szemerédi's Theorem in Terms of Ultrafilters

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Van der Waerden's theorem asserts that if you color the natural numbers with, say, five different colors, then you can always find arbitrarily long sequences of numbers that have the same color and that form an arithmetic progression. Szemer\'edi's theorem generalizes this statement and asserts that every subset of natural numbers with positive density contains arithmetic progressions of arbitrary length.
Van der Waerden's theorem can be proven using elementary combinatorics, but it is also possible to give an interpretation and a short proof in terms of ultrafilters and the Stone-\v{C}ech compactification {\beta}N. This diploma thesis gives an interpretation of Szemer\'edi's theorem in terms of ultrafilters as well. In particular, van der Waerden's theorem is equivalent to the existence of a single ultrafilter with special properties and we will show that Szemer\'edi's theorem is equivalent to the fact that with respect to a counting measure on {\beta}N, almost all ultrafilters have these properties.

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