



Mathematics > Functional Analysis

# Paraproducts via $H^\infty$ -functional calculus

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Let  $X$  be a space of homogeneous type and let  $L$  be a sectorial operator with bounded holomorphic functional calculus on  $L^2(X)$ . We assume that the semigroup  $\{e^{-tL}\}_{t>0}$  satisfies Davies-Gaffney estimates. In this paper, we introduce a new type of paraproduct operators that is constructed via certain approximations of the identity associated to  $L$ . We show various boundedness properties on  $L^p(X)$  and the recently developed Hardy and BMO spaces  $H^p_L(X)$  and  $BMO_L(X)$ . In generalization of standard paraproducts constructed via convolution operators, we show  $L^2(X)$  off-diagonal estimates as a substitute for Calderón-Zygmund kernel estimates. As an application, we study differentiability properties of paraproducts in terms of fractional powers of the operator  $L$ . The results of this paper are fundamental for the proof of a  $T(1)$ -Theorem for operators beyond Calderón-Zygmund theory, which will be the subject of a forthcoming paper.

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