Mathematics > Dynamical Systems

A Brouwer fixed point theorem for graph endomorphisms

Oliver Knill

(Submitted on 4 Jun 2012)

We prove a Lefschetz formula for general simple graphs which equates the Lefschetz number L(T) of an endomorphism T with the sum of the degrees i(x) of simplices in G which are fixed by T. The degree i(x) of x with respect to T is defined as a graded sign of the permutation T induces on the simplex x multiplied by -1 if the dimension of x is odd. The Lefschetz number is defined as in the continuum as the super trace of T induced on cohomology. In the special case where T is the identity, the formula becomes the Euler-Poincare formula equating combinatorial and cohomological Euler characteristic. The theorem assures in general that if L(T) is nonzero, then T has a fixed clique. A special case is a discrete Brouwer fixed point theorem for graphs: if T is a graph endomorphism of a connected graph G, which is star-shaped in the sense that only the zeroth cohomology group is nontrivial, like for connected trees or triangularizations of star shaped Euclidean domains, then there is clique x which is fixed by T. Unlike in the continuum, the fixed point theorem proven here looks for fixed cliques, complete subgraphs which play now the role of "points" in the graph. Fixed points can so be vertices, edges, fixed triangles etc. If A denotes the automorphism group of a graph, we also look at the average Lefschetz number L(G) which is the average of L(T) over A. We prove that this is the Euler characteristic of the graph G/A and especially an integer. We also show that as a consequence of the Lefschetz formula, the zeta function zeta(T,z) is a product of two dynamical zeta functions and therefore has an analytic continuation as a rational function which is explicitly given by a product formula involving only the dimension and the signature of prime orbits of simplices in G.

Comments:	24 pages, 6 figures
Subjects:	**Dynamical Systems (math.DS)**; Discrete Mathematics (cs.DM); General Topology (math.GN)
MSC classes:	58J20, 47H10, 37C25, 05C80, 05C82, 05C10, 90B15, 57M15, 55M20
Cite as:	**arXiv:1206.0782 [math.DS]**
	(or **arXiv:1206.0782v1 [math.DS]** for this version)

Submission history

From: Oliver Knill [view email]
[v1] Mon, 4 Jun 2012 22:13:51 GMT (896kb,D)